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AIR FORCE SURVEYS IN GEOPHYSICS, NO. 193

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L. G. HANSCOM FIELD, BEDFORD, MASSACHUSETTS

A Statistical Model of the Temperature Field at Supersonic Aircraft Altitudes

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Abstract

Data archives are such that, for a selected cruising level, a reasonably large sample of daily average temperatures along a specific route can be obtained and summarized into a frequency distribution. But if, as in Air Force operations, many different and constantly changing routes are involved, then the climatologic tools and methods must be such as to provide estimates of the route average temperature without recourse to the archives and the processing of raw data. A method has been devised that utilizes a model of the horizontal spatial correlations of temperature. It has been demonstrated on a continent-wide area, and shows promise of hemisphere-wide application.

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A Statistical Model of the Temperature Field at Supersonic Aircraft Altitudes

1. INTRODUCTION

Supersonic aircraft (SSA) will cruise at 60000- to 80000-ft altitudes where the ambient air temperature will affect the rate of fuel consumption. It has been estimated (Nelms, 1964), with some qualification (Stickle, 1965), that the allowable payload on a 3000-mile trip will be reduced by perhaps nine passengers and their baggage when the average route temperature at cruising level is 20°F warmer than the U. S. Standard. At the same time it appears that a flying mission will not gain by a diversion from a great-circle route to avoid extremes of route temperature (Serebreny, 1964).

The subject of the average, or integrated, route temperature has not been neglected in the past half-dozen years (Crutcher, 1963; Charles, 1964; Court and Abrahms, 1964). The U. S. Weather Bureau, under FAA sponsorship, issued three special reports on the routes New York to San Francisco (1963), San Francisco to Thule to Stockholm (1964) and New York to Paris (1964), in which there are tables of the distribution of average route temperatures at several key levels for all seasons. The method that the USWB used to obtain the route averages was unequivocally sound. On each of a large sample of days, at each level and season, the average temperature along the selected route was computed. In Figure 1, curve (c) shows the frequency distribution of these averages for the San Franciso to Thule

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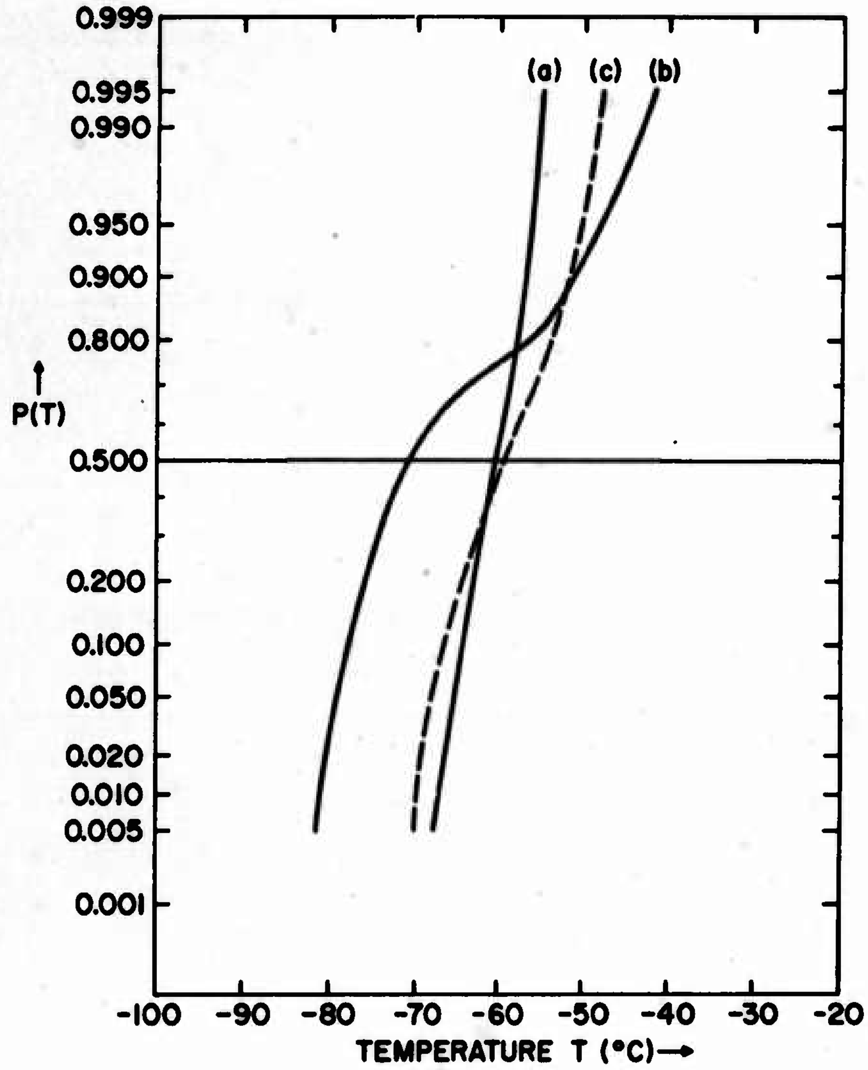

Figure 1. The Cumulative Frequency Distribution P(T) of Temperature T, 50 mb, Winter, Over (a) San Francisco, and (b) Thule; (c) San Francisco — Thule Route Average

route in winter. Boeing Aircraft Co. (Wells, 1962 a, b) has issued two reports on the average route temperature means and standard deviations for many conceivable routes throughout the world. All these tables, however, have to date been limited to the levels of subsonic jet aircraft. Neal Barr, of Boeing, has in a private communication mentioned that a revision that will include the 50-mb level is about to be published.

From the standpoint of Air Force operations, consideration must not be restricted to a set of preselected routes. It must be possible to find the probability of temperature — warm or cold — at relatively short notice for any route and without recourse to the data archives and a computer effort. It has therefore been deemed necessary to devise a model whereby temperatures can be related spatially.

On the assumption that the SSA will fly at a constant pressure height, the model described below has been applied to the North American continent, at levels 50-mb and 30-mb (67500 and 78000 ft) in all four seasons of the year.

2. THE BASIC RELATION

At any given altitude and season, the temperature frequency distribution over a given station generally departs from a normal or gaussian distribution (Barr, 1964). More often than not this departure is small, but sometimes it becomes large, especially over the polar regions at altitudes of 60000 to 80000 ft where the temperature distribution in winter is bimodal, owing to a well-documented phenomenon termed the explosive warming (Reed et al., 1963; Craig and Hering, 1959). When the temperature is averaged over a route of 2000 naut. mi. or longer, however, a centralizing effect produces a more nearly normal distribution. For example, Figure 1 shows the cumulative frequency distribution of the temperature in winter at 50 mb: (a) over San Francisco, (b) over Thule, Greenland. Curve (b) shows distinct bimodality, but curve (c), the distribution of the average over the entire route from San Francisco to Thule, is much less bimodal. For the model described below, all route-integrated temperatures are assumed to be normally distributed and therefore adequately defined by the mean and the standard deviation of the distribution.

Consider a route defined by (n+1) checkpoints $X_o, \ldots, X_n$, including the point of departure X_o and the destination X_n. Let $s_o, \ldots, s_n$, be the weights to be attached to each observation for a suitable determination of route average temperature, and $T_o, \ldots, T_n$, the temperature of each of the (n+1) checkpoints. Then the route average T is given by

$$T = \sum_{i=o}^{n} s_i T_i \Big/ \sum_{i=o}^{n} s_i , \tag{1}$$

to give the mean

$$\overline{T} = \sum_{i=o}^{n} s_i \overline{T}_i \Big/ \sum_{i=o}^{n} s_i . \tag{2}$$

The standard deviation σ of the route average is given in terms of the standard deviation σ_i of each temperature T_i and the correlation ρ_{ij} between the ith and jth terms in the average. Thus,

$$\sigma^2 = \sum_{i,j=0}^{n,n} s_i s_j \rho_{ij} \sigma_i \sigma_j \Big/ \left(\sum_{i=o}^{n} s_i\right)^2 . \qquad (3)$$

When a route is divided into relatively short segments by the checkpoints $X_o, \ldots, X_n$, the weight s_o should be taken as the length from X_o to midway between X_o and X_1, the weight s_1 should be taken as the length from midway between X_o and X_1 to midway between X_1 and X_2, and so on. Table 1 lists the winter 50-mb mean $\overline{T}_i$ and standard deviation σ_i at each of eight checkpoints on the route from San Francisco to Thule. The total distance of 2810 naut. mi. was divided into eight lengths, shown as the weighting factors s_i. The data sample that yielded the means and standard deviations (1958 to 1963) also yielded the correlation coefficients in the table. Thus, Table 1 provides the information necessary and sufficient for estimating mean $\overline{T}$ and standard deviation σ by means of Eqs. (2) and (3).

Table 1. The San Francisco — Thule Route Temperatures, Winter, 50 mb, at Eight Checkpoints. ($\overline{T}_i$ = temperature mean, in °C; σ_i = standard deviation, in °C; s_i = length of the route, in nautical miles, represented by the temperature at the checkpoints. The body of the table contains correlation coefficients between temperatures of pairs of checkpoints.)

Station No. Location	0 SF	1 Medford	2 Seattle	3 Edmonton	4 Ft. Smith	5 Baker Lake	6 Resolute	7 Thule
Mean $\overline{T}_i$	-60	-59	-58	-57	-57	-57	-62	-62
SD σ_i	3. 3	3. 8	4. 1	6. 0	8. 4	8. 8	10. 6	9. 7
Length s_i	150	320	390	390	400	480	470	210
Station No.								
0	1. 000	. 85	. 73	. 43	. 24	. 05	-. 09	-. 27
1		1. 000	. 89	. 63	. 53	. 19	. 00	-. 28
2			1. 000	. 80	. 68	. 22	. 12	-. 26
3				1. 000	. 90	. 66	. 40	. 06
4					1. 000	. 89	. 69	. 38
5						1. 000	. 85	. 62
6							1. 000	. 89
7								1. 000

This technique yielded the means of the average route temperatures and estimated standard deviations in Column B, Table 2. For comparison, the FAA standard deviation values based on 1957 to 1962 data, computed directly from day-to-day route averages, are shown in Column C.

Both Column B and C estimates are made without modeling and are equally valid. The difference is therefore an indication of the error in estimating the route-temperature standard deviation. The root-mean-square difference of the four pairs of values is 0.4°C.

The same technique could be applied to obtain the mean and standard deviation of the route average for any hemispheric route. Success would depend on using a sufficient number of route checkpoints and knowing not only the individual means and standard deviations of each checkpoint but the correlation coefficients between pairs. The correlation coefficients present a difficulty. To adequately cover the Northern Hemisphere, or even the North American quadrant, the tables to give such intercorrelations would have to be voluminous, to say nothing of the massive effort that would be required to produce them. It has therefore been found necessary to develop a model that yields usable estimates of these correlation coefficients.

Table 2. Comparison of Means and Standard Deviations of Route Average Temperatures at 50 mb

Season	Route	Mean (°C)	Standard Deviation (°C)		
			A*	B†	C‡
Summer	SF — NY	-53.5	.98	.96	1.00
	SF — Thule	-48	1.22	1.26	1.17
Winter	SF — NY	-59	2.70	2.62	2.36
	SF — Thule	-59	5.63	5.68	5.48

*by model
†by formula (3)
‡from USWB reports

3. THE MODEL

Figure 2 is an example of the kind of field that a model must be able to approximate. The lines join points having equal correlation coefficients, based on the 50-mb winter temperature at the master station, San Francisco, and the continental slave stations. As might be expected, the correlation close to the master station is high. It decreases to zero at about 1800 naut. mi. from the master station, and becomes slightly negative at greater distances.

After some trial and error, a model with a single parameter (α_{ij}) was chosen to estimate the correlation ρ_{ij} between points X_i and X_j. If the distance between these points is γ_{ij} as measured in earth radians on a polar stereographic chart, then the estimate of ρ_{ij} is

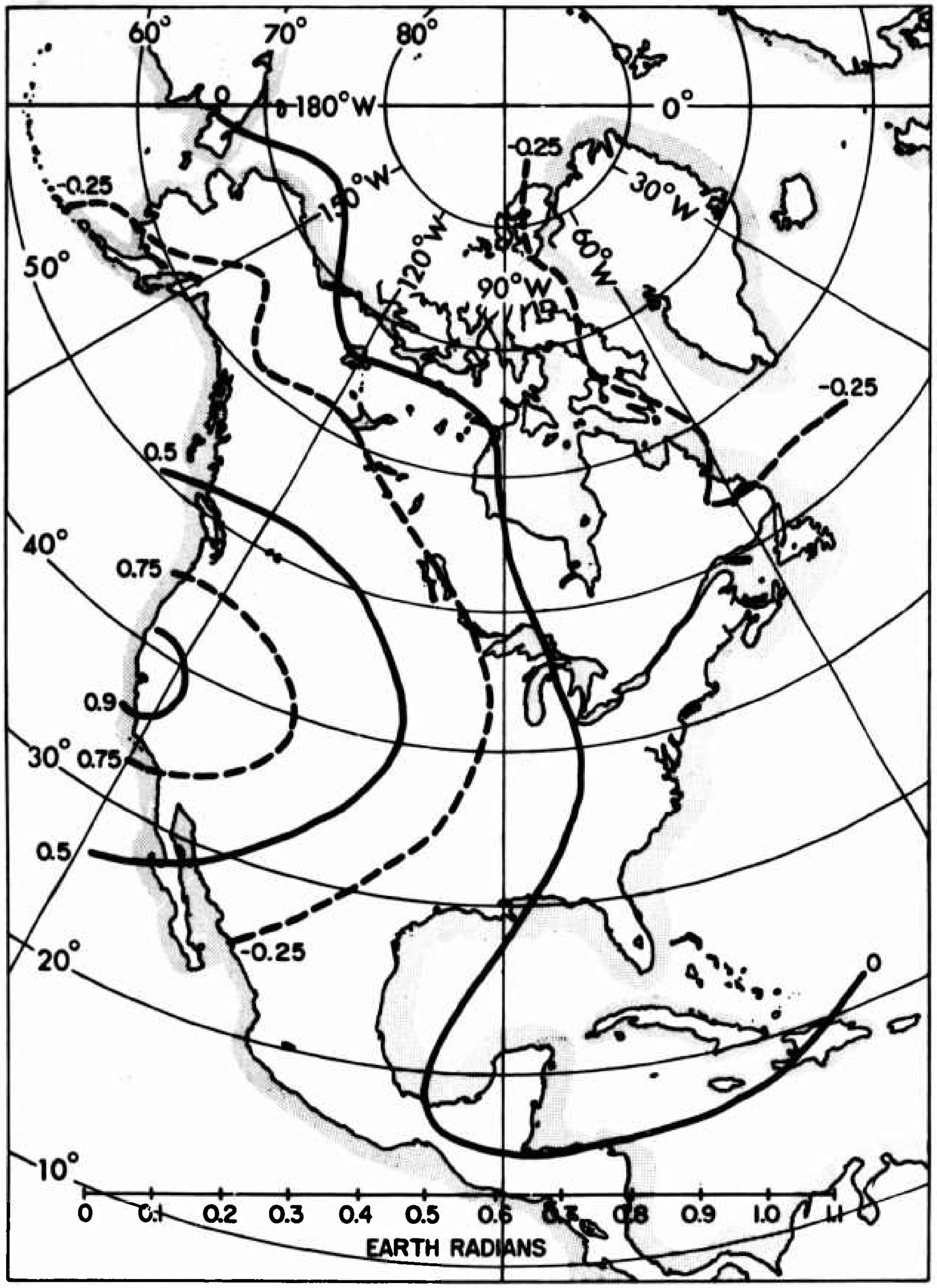

Figure 2. Isopleths of Correlation Coefficient Between the 50-mb Temperature, Winter, Over San Francisco and Other North American Stations [projection: polar stereographic; scale measure: earth radians (tenths)]

$$\hat{\rho}_{ij} = \exp\left[-\frac{1}{2}(\alpha_{ij}\,\gamma_{ij})^2\right]\cos(\alpha_{ij}\,\gamma_{ij})\,, \tag{4}$$

as drawn in Figure 3.

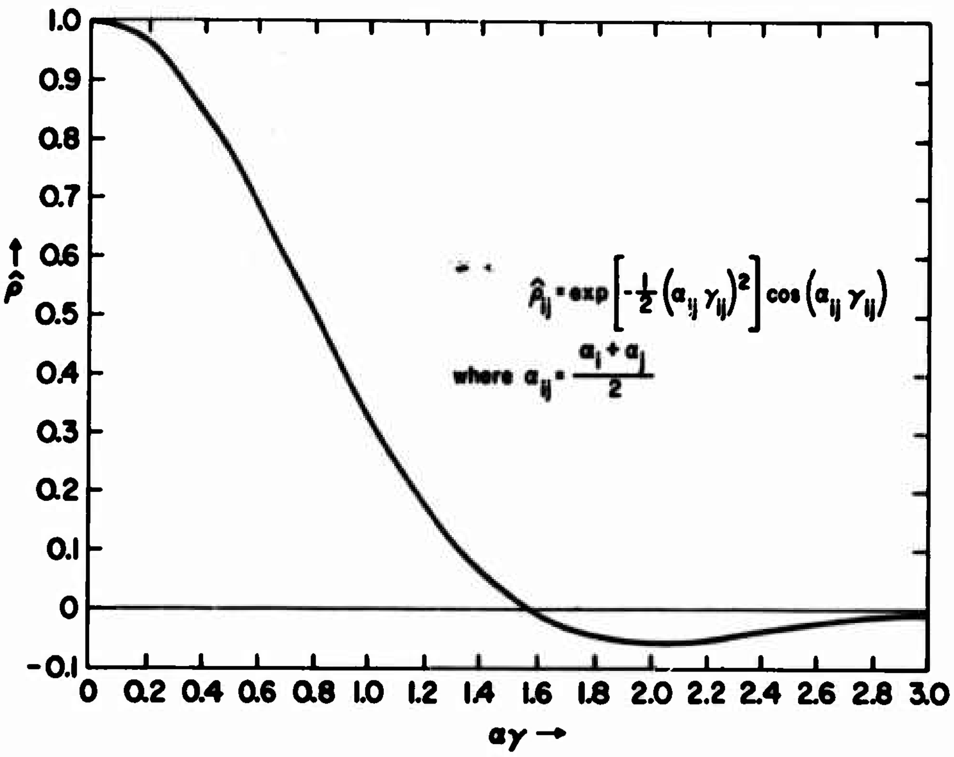

Figure 3. Graph of the Model of Correlation Coefficient (ρ_{ij}) Decay With Distance [γ_{ij} = Distance in Radians Between ith and jth Stations; α_{ij} = Model Parameter]

The model in Figure 3 [Eq. (4)], has merit but it also has limitations. The correlation coefficient will decrease to zero and become somewhat negative; it cannot be more negative than -0.06, but this is not an important consideration when the distance between stations is great. Of greater significance, the more independent the temperature over a given station, the larger the parameter α_{ij}. Since the distance γ_{ij} is made equal to the distance as measured on a polar stereographic projection instead of equal to the actual terrestrial distance between stations, the effect of the varying scale is absorbed in the parameter α_{ij}.

To construct the model, let the origin on a polar stereographic chart be at the N pole, the x axis positive along the 0° meridian, and the y axis positive along the 90°E meridian. Let λ denote longitude and ϕ denote latitude. Then the distance γ_{ij} between the two points (x_i, y_i) and (x_j, y_j), which respectively represent the two points (λ_i, ϕ_i) and (λ_j, ϕ_j), is given by

$$\gamma_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \; , \tag{5}$$

where

$$x_i = 2 \cos \lambda_i \tan\left(\frac{\pi}{4} - \frac{\phi_i}{2}\right) ,$$

$$y_i = 2 \sin \lambda_i \tan\left(\frac{\pi}{4} - \frac{\phi_i}{2}\right) .$$

Likewise,

$$x_j = 2 \cos \lambda_j \tan\left(\frac{\pi}{4} - \frac{\phi_j}{2}\right) ,$$

$$y_j = 2 \sin \lambda_j \tan\left(\frac{\pi}{4} - \frac{\phi_j}{2}\right) .$$

Thus, the radial distance from pole to equator, which is $\pi/2$ radians on the earth's surface, is 2.0 radians on the polar stereographic chart. Figure 2 contains a constant scale for the chart, marked off in tenths of radians (see Table 3).

Table 3. Construction of Scale Yielding Segments of Equal Length on the Polar Stereographic Chart [All measurements are made from the N pole along the 0° meridian (x axis).]

$y = 0;\ x = 2 \tan(\frac{\pi}{4} - \frac{\phi}{2})$	
x	ϕ(°N)
0.1	84.2
0.2	78.6
0.3	67.4
0.4	67.4
0.5	62.0
0.6	56.6
0.7	51.4
0.8	46.4
0.9	41.6
1.0	36.8

At this stage the model does not appear especially useful. The parameter α_{ij} is a function of the two stations X_i and X_j. What is needed is a single parameter that can be assigned to each station individually so that it can be mapped in much the same way as the mean or standard deviation is mapped.

Arbitrarily assign the number α_i to the station and make the parameter α_{ij} the average of two values:

$$\alpha_{ij} = \frac{(\alpha_i + \alpha_j)}{2} \tag{6}$$

But this arbitrary settlement of the question leads to another question: What is the best value of α_i to assign to station X_i? Clearly, it should be a value that minimizes the error in estimation of ρ_{ij} by Eqs. (4) and (6).

To attack the problem expeditiously it was decided to find the value of α_i at each master station X_i so that when α_i was used in place of α_{ij} in Eq. (4), the result would be a set of estimates of ρ_{ij} between the ith station and all other stations within a reasonable distance of the ith station. The value of α_i was chosen such that

$$E_i = \sum_{j=1}^{65} w_{ij} (\rho_{ij} - \hat{\rho}_{ij})^2 \tag{7}$$

was a minimum, where E is the weighted sum of squares of error. The quantity w_{ij} is a weighting factor. It can be seen from Eq. (3) and its application in Table 1 that it is more important to closely estimate the correlations of neighboring stations than to estimate the correlations of stations farther apart. After a few trials, it was decided to set the weighting factor

$$w_{ij} \begin{cases} = 0.6 - \gamma_{ij}, & \gamma_{ij} < 0.6, \\ = 0, & \gamma_{ij} \geq 0.6. \end{cases} \tag{8}$$

4. DATA AND RESULTS

To apply the model, the semidaily temperatures for the five years 1959 to 1963 were assembled for the 50- and 30-mb heights at which SST are expected to cruise. The data, from 66 stations over North and Central America (Table 4), were grouped

Table 4. The Sixty-six Stations Whose Observations of the 50-mb and 30-mb Temperatures for the Period of Record, December 1958 to November 1963, Were Correlated for This Report

Sta. No.	Location	Latitude	Longitude
1	San Diego, California	32 49	117 08
2	Balboa, Panama, Canal Zone	8 59	079 34
3	San Juan, Puerto Rico	18 26	066 00
4	Grand Cayman, BWI/Roberts Fld	19 18	081 22
5	San Andres, Colombia	12 35	081 42
6	Miami, Florida	25 48	080 16
7	Burrwood, Louisiana	28 58	089 22
8	Merida, Mexico	20 55	089 40
9	Brownsville, Texas	25 54	097 26
10	Greensboro, North Carolina	36 05	071 57
11	Charleston, South Carolina	32 54	080 02
12	Nashville, Tennessee	36 07	086 41
13	Fort Worth, Texas/Carswell AFB	32 50	097 03
14	Jackson, Mississippi	32 20	090 13
15	Dodge City, Kansas	37 46	099 58
16	Sable Island, Nova Scotia	43 56	060 02
17	Nantucket, Massachusetts	41 15	070 04
18	Portland, Maine	43 39	070 19
19	Peoria, Illinois	40 40	089 41
20	Sault Ste Marie, Michigan	46 28	084 22
21	Goose, Newfoundland	53 18	060 27
22	Fort Chimo, Quebec	58 06	068 26
23	Nitchequon, Quebec	53 12	070 54
24	Fort Harrison, Quebec	58 27	078 08
25	Moosonee, Ontario	51 16	080 39
26	Trout Lake, Ontario	53 50	089 52
27	Churchill, Manitoba	58 45	094 04
28	Frobisher Bay, Northwest Territory	63 45	068 33
29	Hall Lake, Northwest Territory	68 47	087 15
30	Baker Lake, Northwest Territory	64 18	096 00
31	Thule, Greenland	76 33	068 49
32	Resolute Bay, Northwest Territory	74 43	094 59
33	Alert, Northwest Territory	82 30	062 20
34	Eureka Sound, Northwest Territory	80 00	085 50
35	Del Rio, Texas	29 23	100 46
36	Midland, Texas/Sloan Fld	32 56	102 12
37	El Paso, Texas	31 48	106 24
38	Grand Junction, Colorado	39 07	108 32
39	Ely, Nevada	39 17	114 51
40	Tucson, Arizona	32 07	110 56
41	Oakland, California	37 44	122 12
42	Bismarck, North Dakota	46 46	100 45
43	North Platte, Nebraska	41 08	100 41
44	Great Falls, Montana	47 29	111 21
45	Medford, Oregon	42 22	122 52
46	Olympia, Washington	47 27	122 18
47	The Pas, Manitoba	53 58	101 06
48	Edmonton, Alberta	53 34	113 31
49	Prince George, British Columbia	53 50	122 41
50	Fort Nelson, British Columbia	58 50	122 35
51	Annette, Alaska	55 02	131 34
52	Yakutat, Alaska	59 31	139 40
53	Cold Bay, Alaska	55 12	162 43
54	St Paul Island, Alaska	57 09	170 13
55	Fort Smith, Northwest Territory	60 01	111 58
56	Coppermine, Northwest Territory	67 49	115 05
57	Anchorage, Alaska	61 10	149 59
58	Fairbanks, Alaska	64 49	147 52
59	Bethel, Alaska	60 47	161 48
60	Nome, Alaska	64 30	165 26
61	Isachsen, Northwest Territory	78 47	103 32
62	Mould Bay, Northwest Territory	76 14	119 20
63	Barter Island, Alaska	70 08	143 38
64	Barrow, Alaska	71 18	156 07
65	Shemya, Alaska	52 43	174 06
66	Pittsburgh, Pennsylvania	40 30	080 13

by seasons (winter: December, January, February; summer: June, July, August). For each station in each season the sample mean and standard deviation of the temperature were found and the correlation coefficient between pairs of all stations calculated. The means and standard deviations were then plotted and isopleths drawn (Figures 4 to 11, a, b). Equation (4) was used in making the estimate $\hat{\rho}_{ij}(\alpha_i)$ and the error of estimate

$$[\rho_{ij} - \hat{\rho}_{ij}(\alpha_i)]$$

was squared, multiplied by w_{ij} and added with other squared errors to give, for the *i*th station:

$$E_i = \sum w_{ij} [\rho_{ij} - \hat{\rho}_{ij}(\alpha_i)]^2 \tag{9}$$

By trying a set of values for α_i,

$$\alpha_i = 0.5, 1.5, \ldots, 10.5,$$

it was possible to find the value of α_i that minimized E_i. The results for α are shown in Figures 4 to 11 (c).

In determining the sample mean $\bar{T}_i$, standard deviation σ_i, and the correlation coefficient ρ_{ij}, between stations, the diurnal cycle was ignored. To find out how serious an oversight this could be, the 0000Z and 1200Z temperatures were averaged separately and their difference $D(D = \bar{T}_{1200} - \bar{T}_{0000})$ plotted for the winter and summer seasons at 50 mb (Figures 12 and 13). The effect of the daytime heating at the 50-mb level is apparent. Close to the 90°W meridian the difference is zero; to the east it is positive and to the west it is negative. It is not, however, clear whether the effect is real or due to heating of the instrument. This subject has been dealt with in some detail by Chiu (1959), Badgley (1959), and Harris et al. (1962).

Between the standard deviations of 0000Z and 1200Z observations, there is a root-mean-square difference of 0.124°C in summer and 0.272°C in winter. Errors of observation should increase the observed variances by a mean-square amount

$$\epsilon^2 = \sigma^2_{(observed)} - \sigma^2_{(true)} \tag{10}$$

The value of ϵ has been found to be (Johannessen, 1959) approximately 0.7. By comparison, the root-mean-square difference between 0000Z and 1200Z observations can be considered negligible, whether it is real or due to errors of the instrument.

For the present study, since the standard deviation is of the order of 2° to 10°C, all errors of observation are considered to have only slight effect on the values of the standard deviation and are therefore neglected. Eventually, refined observations and larger samples of data could be used to yield improved values of means, standard deviations, and correlation coefficients, so that the charts of this paper might be improved. There are similarities, but also significant differences, between the charts in this report and the charts or tables of Ratner (1957), Air Weather Service (1953), Li Peng (1965), and Smith, McMurray, and Crutcher (1963). All the charts differ from one another in significant values but the 50-mb and 30-mb averages given by Ratner from the 1946-1955 records are in satisfactory agreement with Figures 4a to 11a in this report.

5. APPLICATION

From the (a) and (b) charts of Figures 4 to 11 we read the mean and standard deviation of temperatures at checkpoints along a selected route. From the (c) charts of Figures 4 to 11 we read the value of α at each checkpoint, and then obtain the correlation coefficients between checkpoints. This provides the requisite information for obtaining the mean and standard deviation of the route average. The results obtained for the San Francisco — Thule route, winter, 50 mb (Table 5), illustrate the procedure.

The chart route is divided into equal segments 0.1 radian long (Table 3). For the San Francisco – Thule route this calls for nine segments, the last one somewhat overshooting Thule. Table 5 gives the means $\overline{T}_i$ from Figure 4a, the standard deviations σ_i from Figure 4b and the parameters α_i from Figure 4c. Since the checkpoints are evenly spaced on the chart, each weight s_i is made inversely proportional to the scale factor, that is,

$$s_i = 1 + \sin\phi_i . \tag{11}$$

The end points X_o and X_9 are given half this weight. For each pair of points,

$$\alpha_{ij} = \frac{(\alpha_i + \alpha_j)}{2} ,$$

$$\gamma_{ij} = \frac{(j - i)}{10} \text{ radians,}$$

Table 5. The Straightline Route on the Polar Stereographic Chart, San Francisco to Thule, Winter, 50 mb. (Segment divisions are 0.1 earth radian in length. For each checkpoint the table shows latitude ϕ_i, in °N; mean temperature $\overline{T}_i$, in °C; standard deviation σ_i, in °C; weighting factor s_i, which is inversely proportional to the scale factor; and parameter α_i. The terms α_{ij}, ρ_{ij}, and $s_i s_j \sigma_i \sigma_j$, in that order, are listed in the body of the table.)

Station No.	0	1	2	3	4	5	6	7	8	9
Latitude ϕ_i	38	42	47	52	57	62	66	71	75	78
Mean $\overline{T}_i$	-60	-59	-58	-57	-57	-57	-57	-58	-62	-64
SD σ_i	3.3	3.7	4.2	5.0	6.2	7.4	8.5	10.0	10.5	9.8
s_i	0.8	1.7	1.7	1.8	1.8	1.9	1.9	1.9	2.0	1.0
α_i	2.5	2.5	2.5	2.4	2.2	2.0	2.3	2.3	2.4	2.4
Station No.										
0	1.00 6.8	2.5 0.93 16.4	2.5 .77 18.5	2.4 .58 23.4	2.4 .36 29.1	2.3 .20 36.7	2.3 .05 42.1	2.4 -.03 49.4	2.4 -.05 54.6	2.4 -.05 25.5
1		1.00 39.7	2.5 .93 44.7	2.4 .74 56.7	2.3 .61 70.6	2.3 .40 88.8	2.4 .18 102.1	2.4 .05 119.7	2.4 -.03 132.3	2.4 -.05 61.7
2			1.00 50.4	2.4 .94 63.9	2.4 .78 79.5	2.3 .61 100.1	2.4 .36 115.0	2.4 .18 134.9	2.4 .05 149.1	2.4 -.03 69.6
3				1.00 81.8	2.3 .95 100.8	2.2 .82 126.9	2.4 .58 145.8	2.4 .36 171.0	2.4 .18 189.0	2.4 .05 88.2
4					1.00 125.4	2.1 .95 157.9	2.2 .82 181.4	2.2 .64 212.8	2.3 .40 235.2	2.3 .21 109.8
5						1.00 198.8	2.2 .95 228.4	2.2 .82 267.9	2.2 .63 296.1	2.3 .43 138.1
6							1.00 262.4	2.3 .95 307.8	2.4 .78 340.2	2.4 .58 158.8
7								1.00 361.0	2.4 .94 399.0	2.4 .79 186.2
8									1.00 441.0	2.4 .94 205.8
9										1.00 96.0

from which it is possible to obtain $\hat{\rho}_{ij}$ from Figure 3 or Eq. (4). In Table 5 the values of $\hat{\rho}_{ij}$ are shown together with the associated values of $s_i s_j \sigma_i \sigma_j$. Finally, the route variance σ^2 is obtained by cumulative multiplication:

$$\sigma^2 = \left[\sum_{i=o}^{9} s_i^2 \sigma_i^2 + 2\sum_{i \neq j} s_i s_j \sigma_i \sigma_j \hat{\rho}_{ij}\right] \Big/ \left(\sum_i s_i\right)^2 . \tag{12}$$

Table 2 shows the standard deviations estimated according to the model (Column A). The root-mean-square difference between these estimates and estimates of the FAA is 0.4°C, the same as the difference between the estimates of Columns B and C, the latter two being equally valid.

To test the efficiency of the assumption that the route average is normally distributed, the 1-percentiles and 5-percentiles were computed by the formula

$$\hat{T} = \bar{T} + y\sigma, \tag{13}$$

where $y = 2.33$ for the 1-percentile, $y = 1.645$ for the 5-percentile. This estimate was compared with the estimates given by the distributions in the tables of the FAA studies. (The latter distributions are actual distributions independent of assumptions about their nature.) The results are shown in Table 6. The root-mean-square difference is 1.1°C for the 1-percentile and 0.1°C for the 5-percentile.

Table 6. Comparison of Two Estimates of the 1-Percentile and 5-Percentile of Average Route Temperature

Season	Route	1-Percentile		5-Percentile	
		A*	B†	A*	B†
Summer	SF — NY	-51.2	-52.1	-51.9	-52.5
	SF — Thule	-45.2	-45.0	-46.0	-45.6
Winter	SF — NY	-52.7	-53.9	-54.5	-55.3
	SF — Thule	-46.0	-47.6	-49.8	-49.5

*by model
†from USWB-FAA studies

6. SUMMARY

To obtain the frequency distribution of the route average temperature on a constant-pressure surface, it is assumed that the route average has a normal gaussian distribution. The mean of the route average is given by:

$$\bar{T} = \sum_{i=o}^{n} s_i \bar{T}_i \Big/ \sum_{i=o}^{n} s_i \ , \qquad \text{[Eq. (2)]}$$

where there are (n+ 1) checkpoints along the route, including the point of departure and destination. The mean temperature at each checkpoint is $\bar{T}_i$ and the weight given to it, s_i, is made directly proportional to the fraction of the route over which $\bar{T}_i$ is a representative temperature. In this paper (see page 12),

$$s_i = 1 + \sin\phi_i \ , \qquad \text{[Eq. (11)]}$$

where ϕ_i is the latitude of the ith checkpoint. The variance of the route average is given by [see Eq. (12)]

$$\sigma^2 = \sum_{i,j=o}^{n,n} s_i s_j \sigma_i \sigma_j \hat{\rho}_{ij} \Big/ \left(\sum_{i=o}^{n} s_i \right)^2 \ ,$$

where σ_i is the standard deviation at the ith checkpoint, and $\hat{\rho}_{ij}$ is the correlation coefficient given by

$$\hat{\rho}_{ij} = \exp\left[- \frac{1}{2}(\alpha_{ij} \gamma_{ij})^2\right] \cos(\alpha_{ij} \gamma_{ij}) \ . \qquad \text{[Eq. (4)]}$$

The distance γ_{ij} in this paper is the chart distance, in earth radians, given by

$$\gamma_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \ , \qquad \text{[Eq. (5)]}$$

where

$$x_i = 2\cos\lambda_i \tan\left(\frac{\pi}{4} - \frac{\phi_i}{2}\right) \ ,$$

$$y_i = 2\sin\lambda_i \tan\left(\frac{\pi}{4} - \frac{\phi_i}{2}\right) \ ,$$

where (ϕ_i, λ_i) are the latitude and longitude of the ith checkpoint. The parameter α_{ij} is an average, that is,

$$\alpha_{ij} = \frac{\alpha_i + \alpha_j}{2} \qquad \text{[Eq. (6)]}$$

where α_i is the parameter value at the ith station, depicted in the (c) charts of Figures 4 to 11.

7. CONCLUSIONS

The sample application shows that the model, consisting primarily of a parameter to obtain spatial correlation of temperature on a constant-pressure surface, is an efficient tool for estimating the standard deviation of the route average temperature and an effective one for estimating the 5- or 1-percentiles. There are refinements that could have been tried but would have been unnecessary. For example, the correlation coefficient might decay more rapidly in a N-S direction than in an E-W direction, hence making it desirable for the value of the parameter α to be a vector combination of two other values (say α_n, α_w). Since the simpler model with only a single parameter per station works so well, attempts at refinements were abandoned.

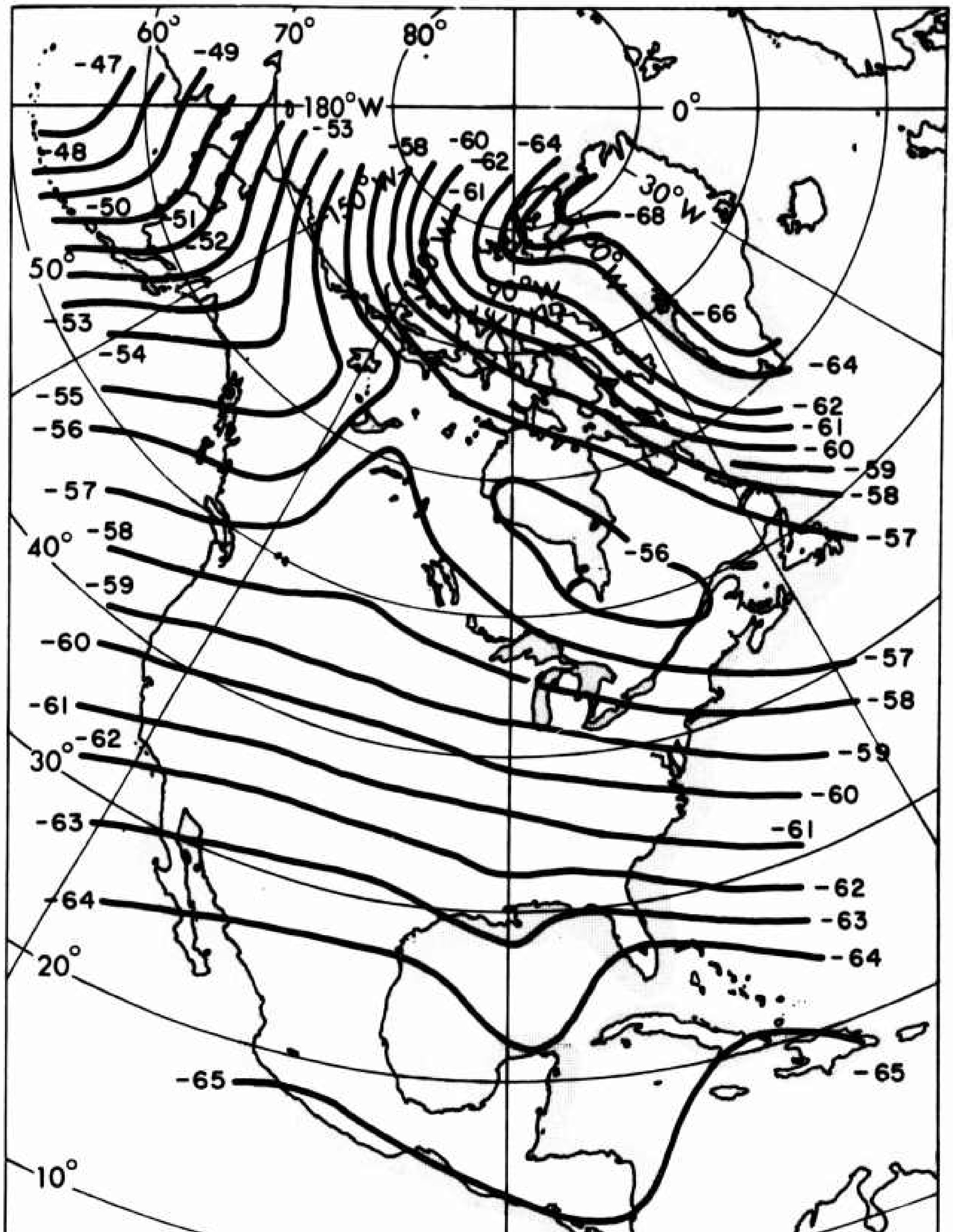

Figure 4a. Isotherms of Mean Temperature $\overline{T}_i$, 50 mb, December, January, February

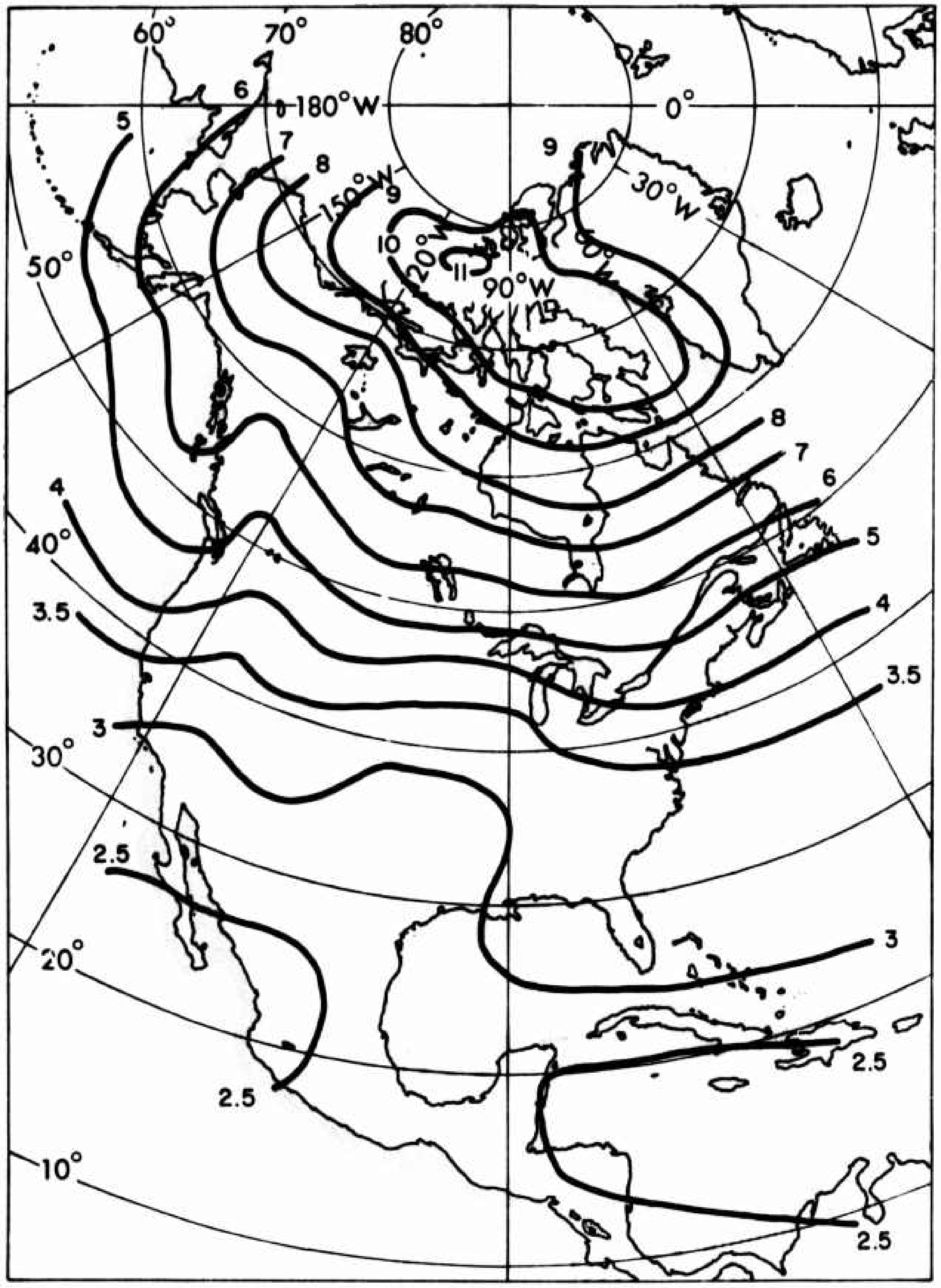

Figure 4b. Isopleths of Standard Deviation σ_i of the Temperature at 50 mb, December, January, February

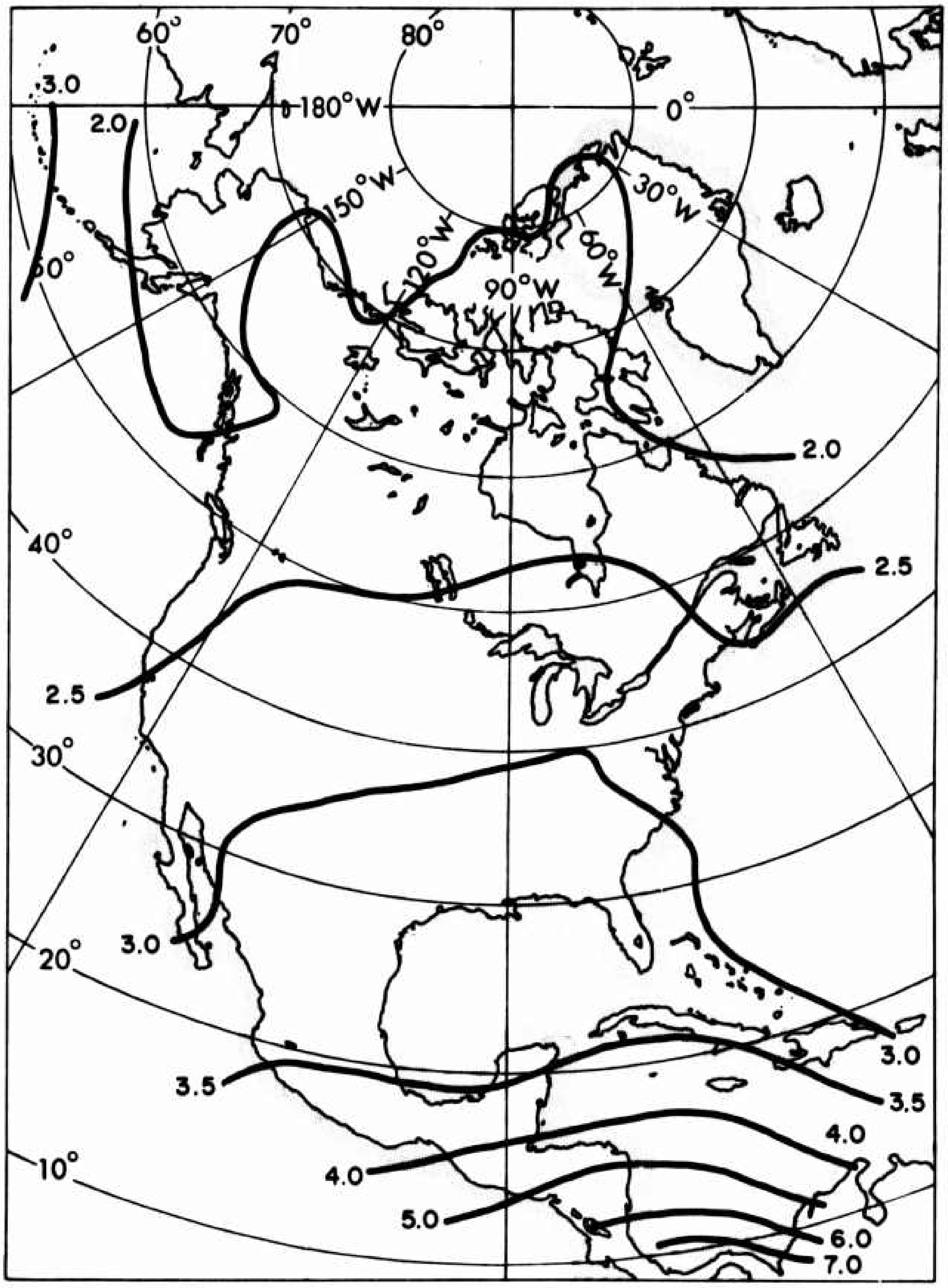

Figure 4c. Isopleths of the Model Parameter α_i, 50 mb, December, January, February

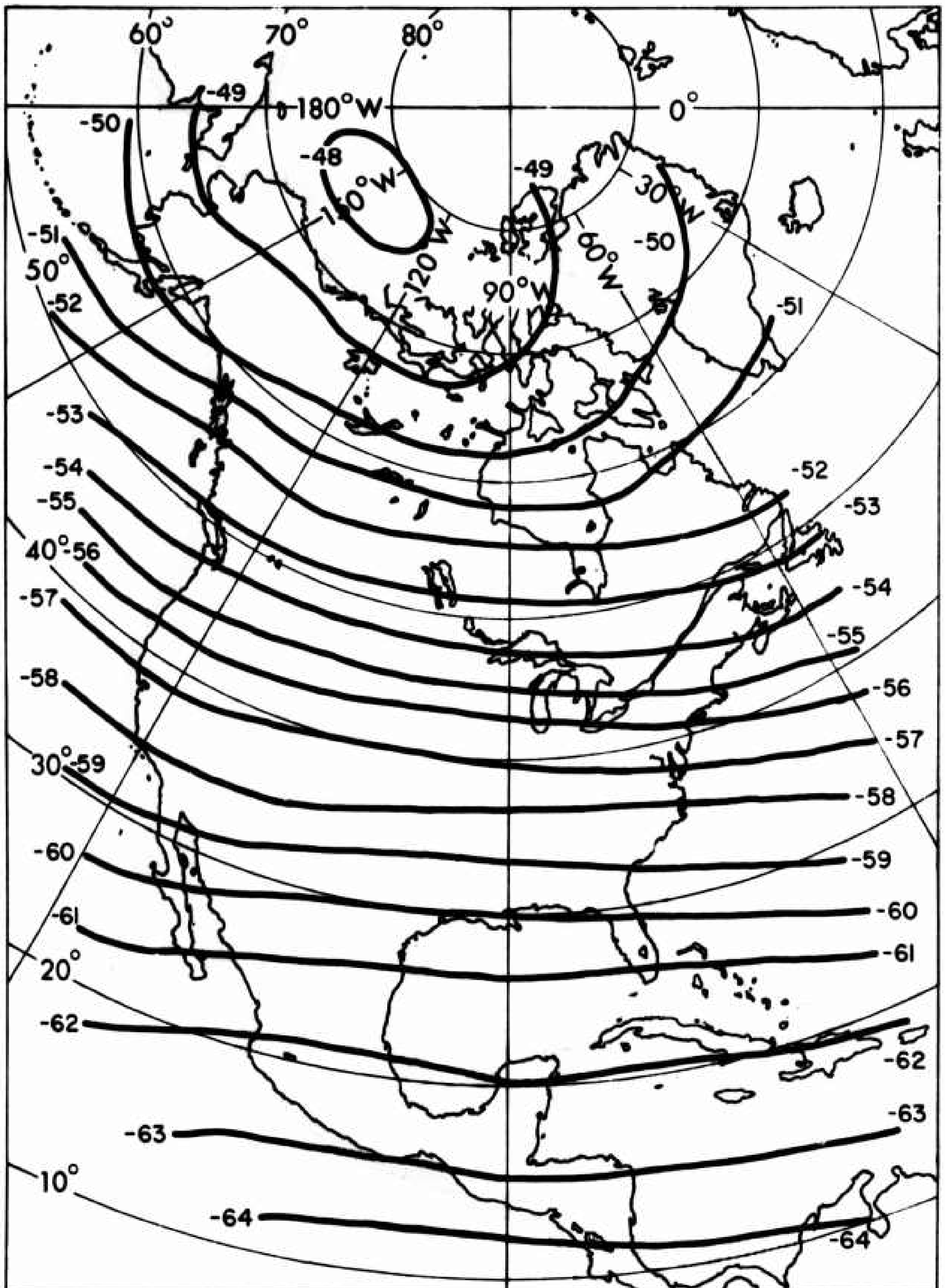

Figure 5a. Isotherms of Mean Temperature $\overline{T}_i$, 50 mb, March, April, May

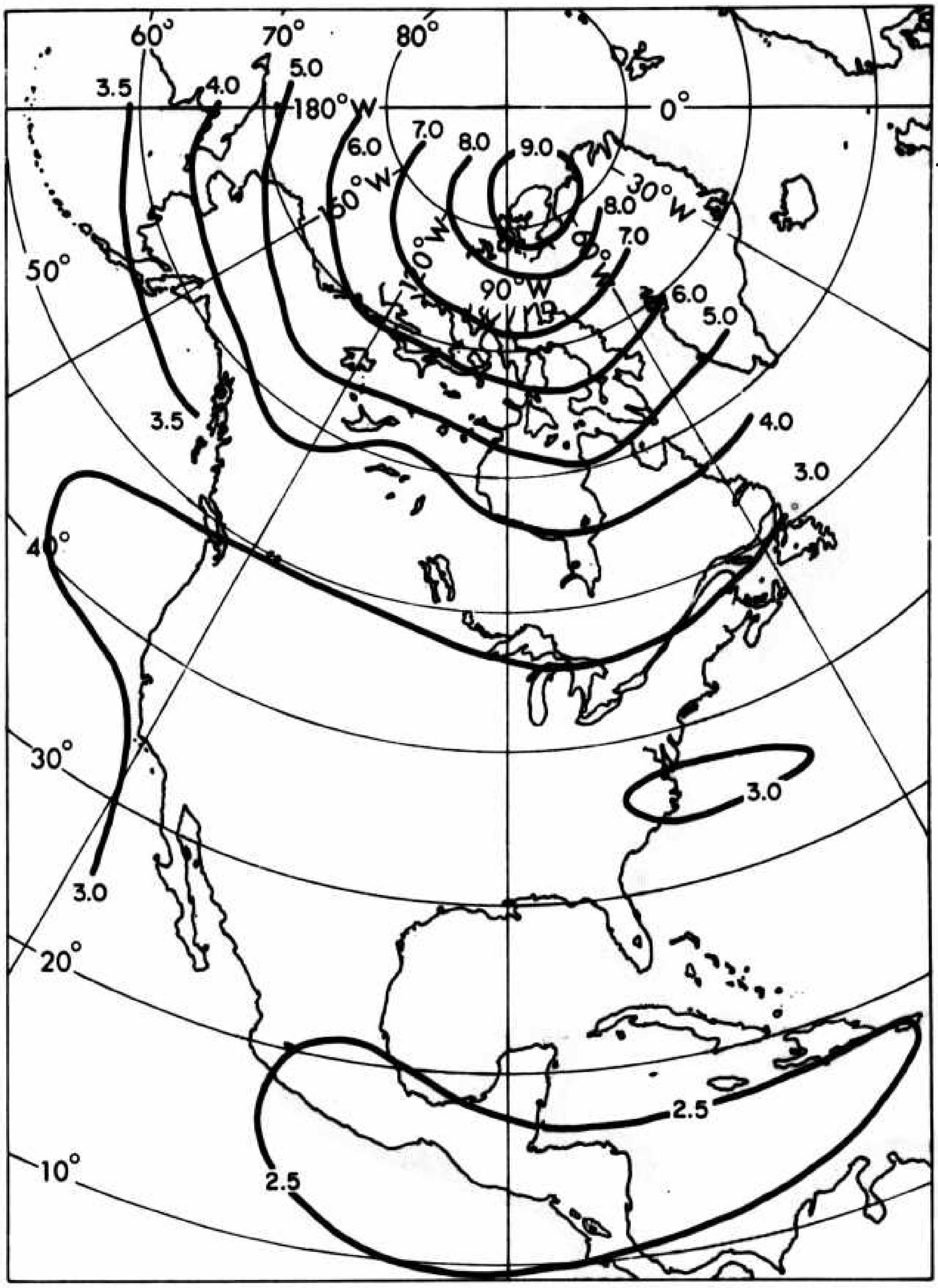

Figure 5b. Isopleths of Standard Deviation σ_i, 50 mb, March, April, May

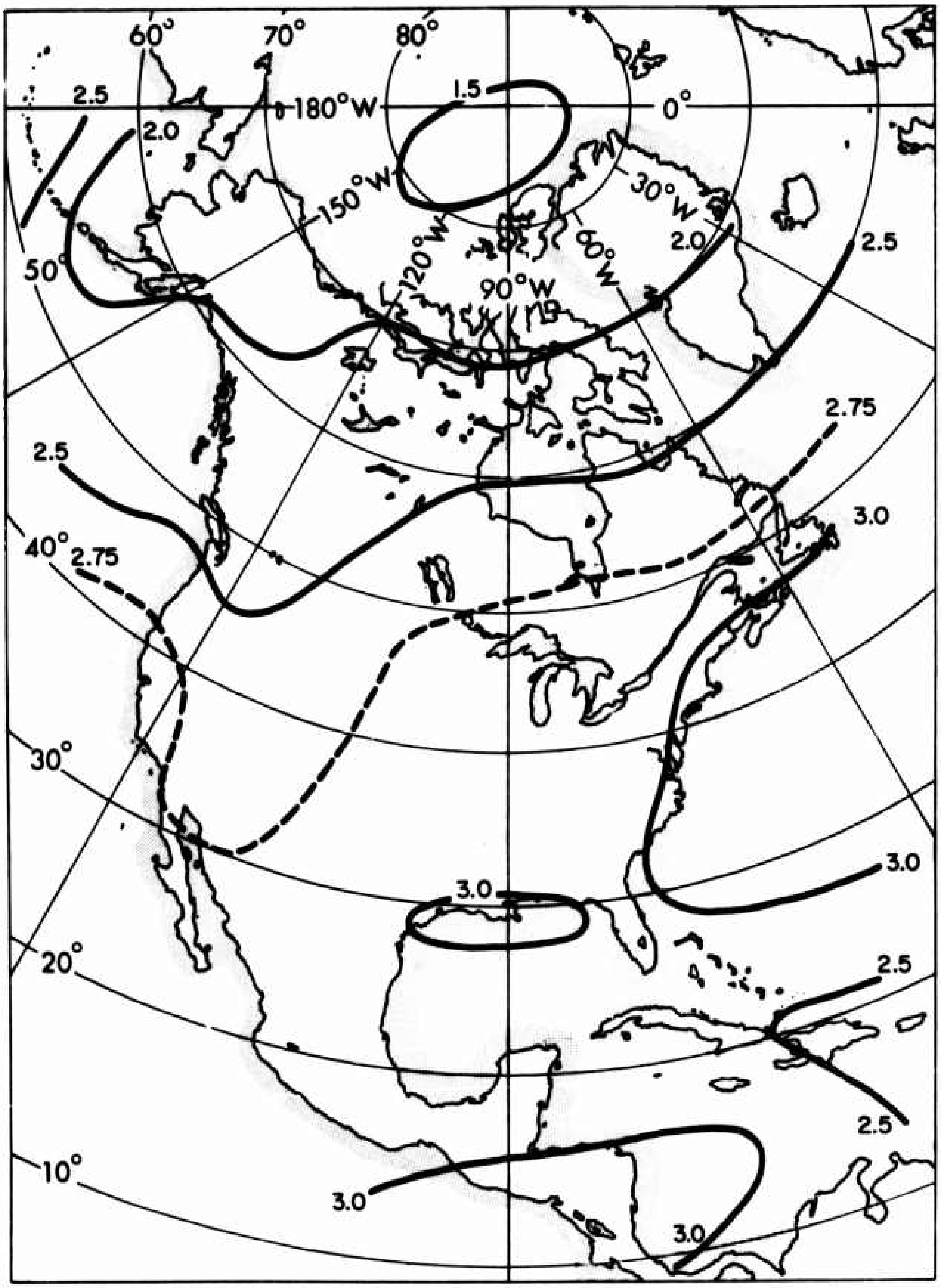

Figure 5c. Isopleths of the Model Parameter α_i, 50 mb, March, April, May

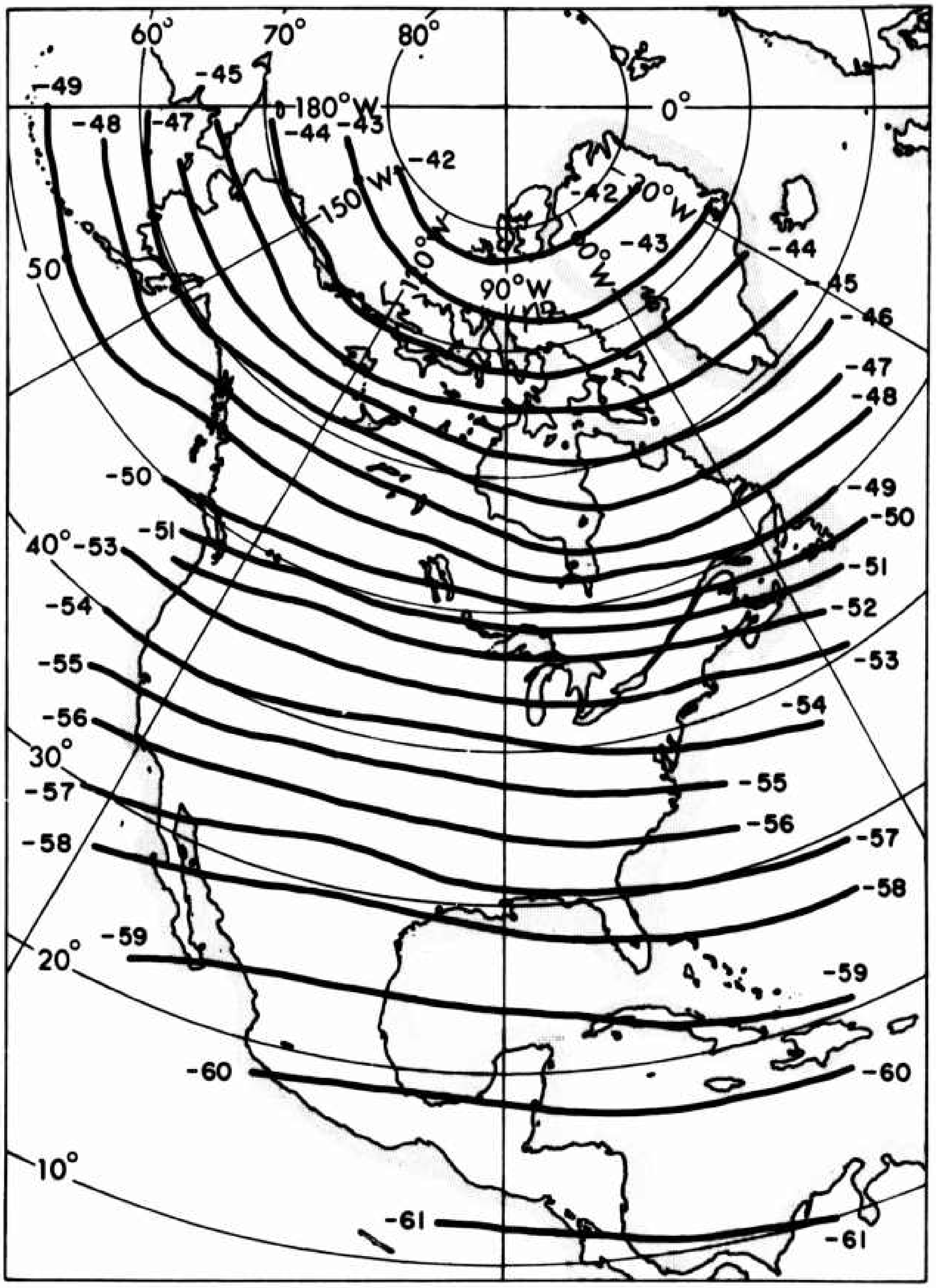

Figure 6a. Isotherms of Mean Temperature $\overline{T}_i$, 50 mb, June, July, August

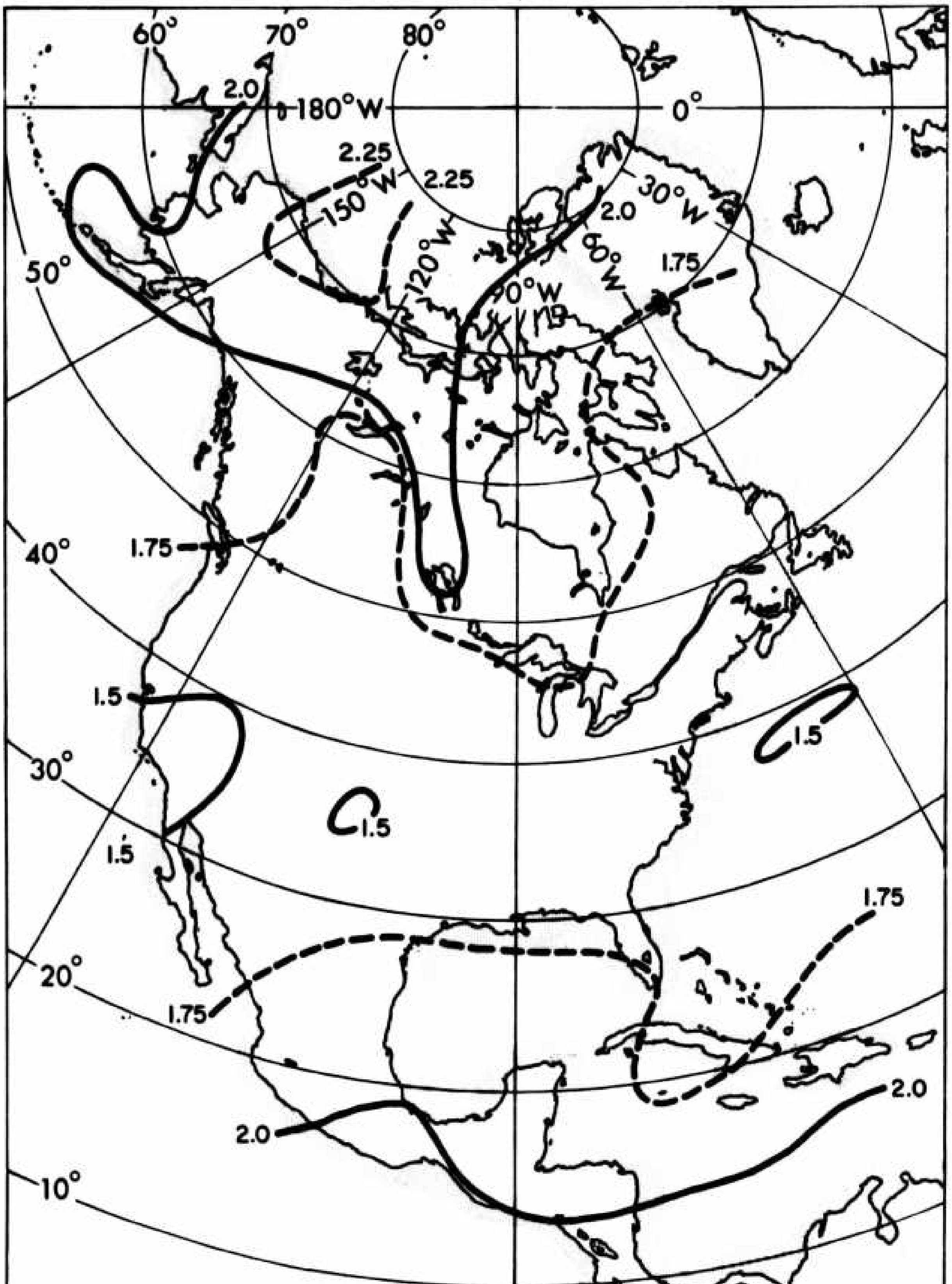

Figure 6b. Isopleths of Standard Deviation σ_i, 50 mb, June, July, August

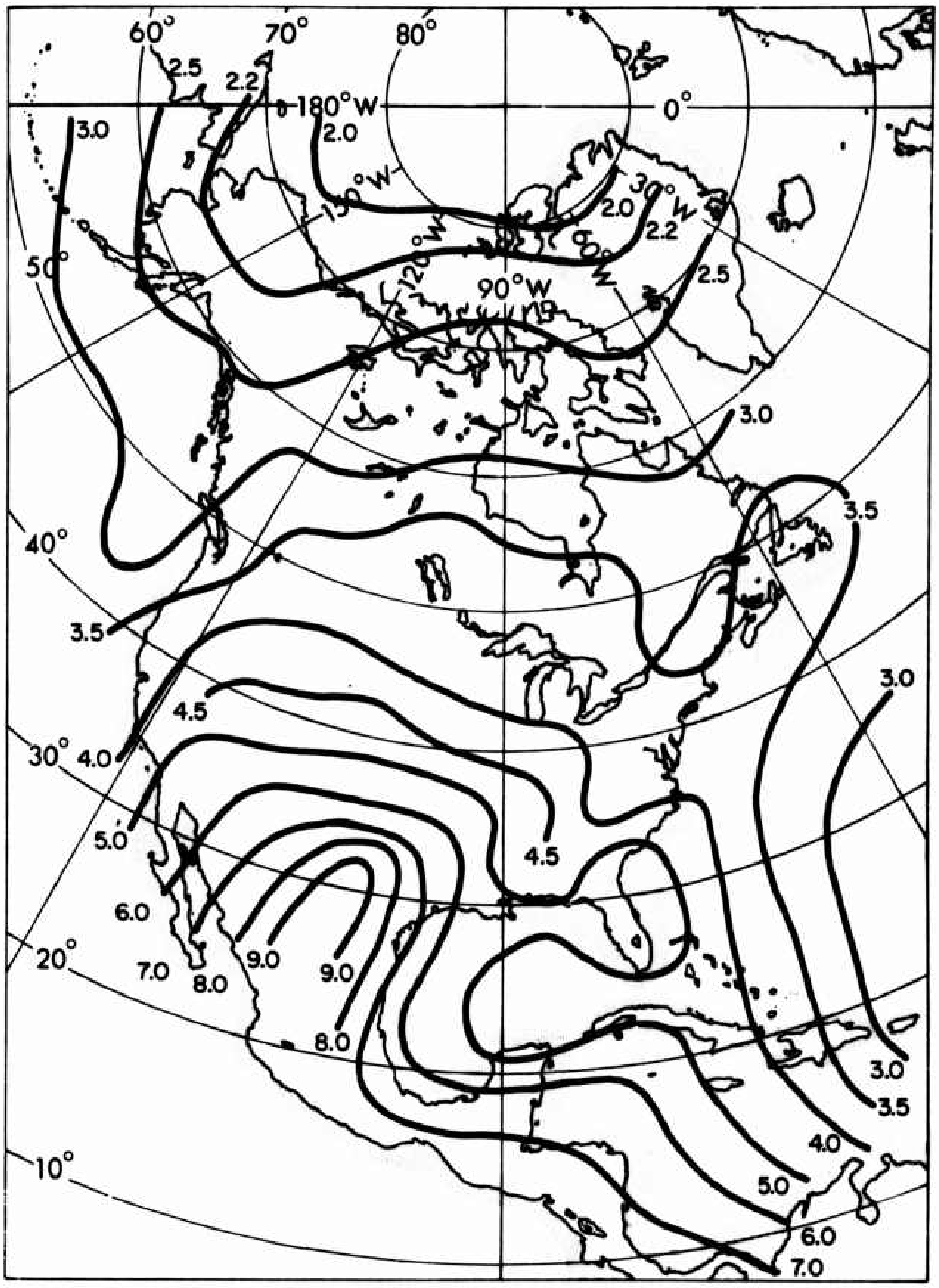

Figure 6c. Isopleths of the Model Parameter α_i, 50 mb, June, July, August

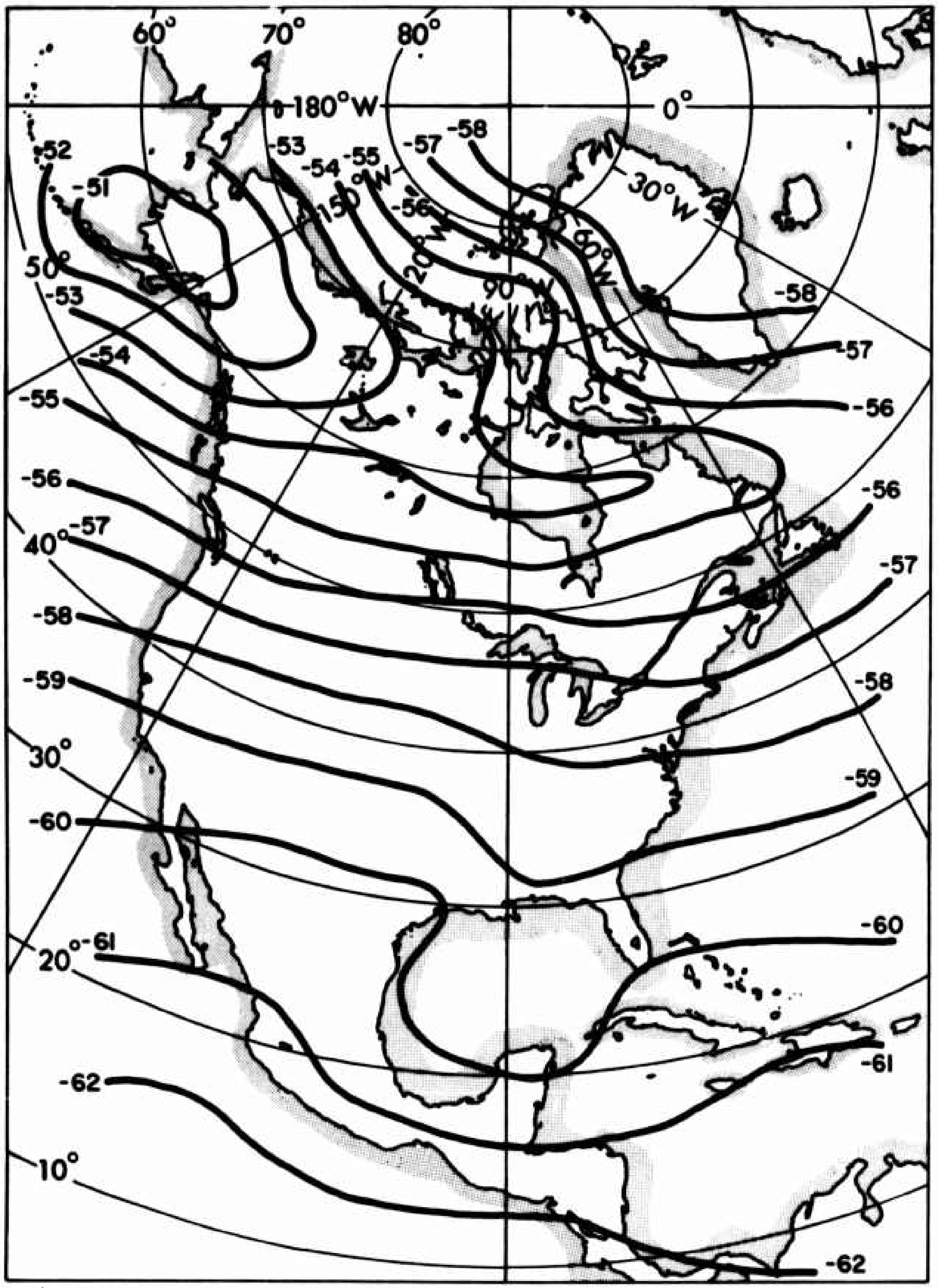

Figure 7a. Isotherms of Mean Temperature $\overline{T}_i$, 50 mb, September, October, November

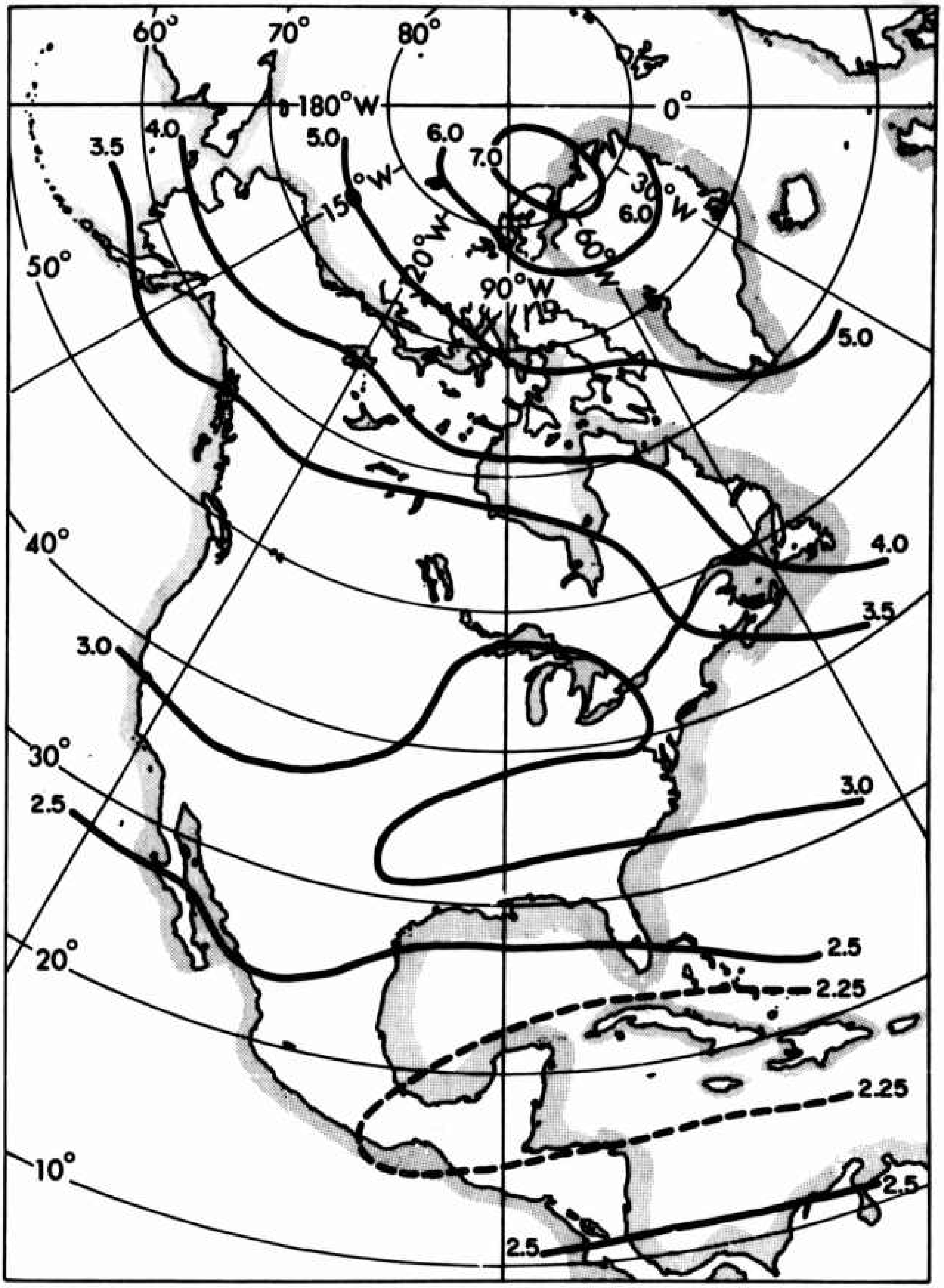

Figure 7b. Isopleths of Standard Deviation σ_i, 50 mb, September, October, November

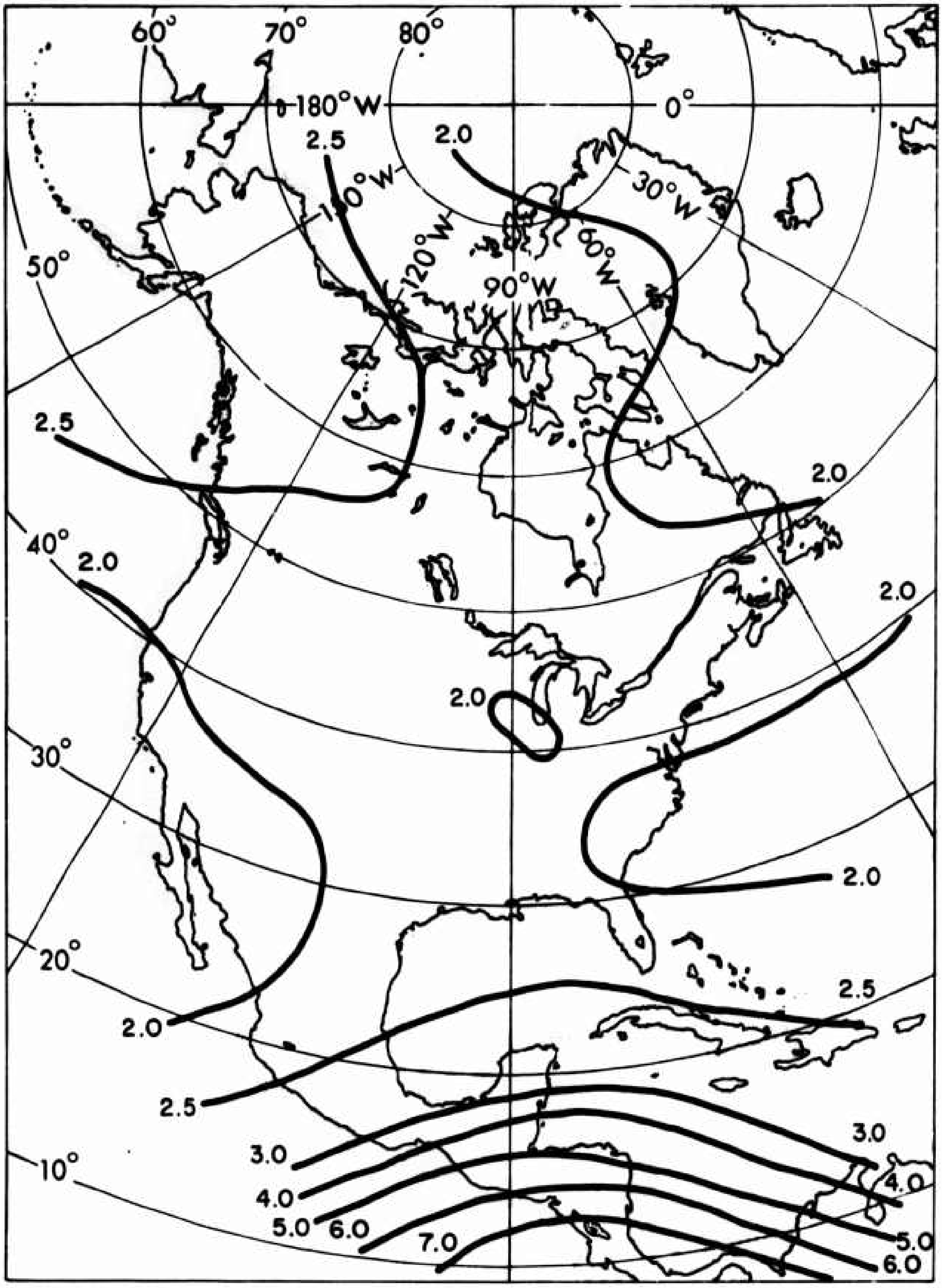

Figure 7c. Isopleths of Model Parameter α_i, 50 mb, September, October, November

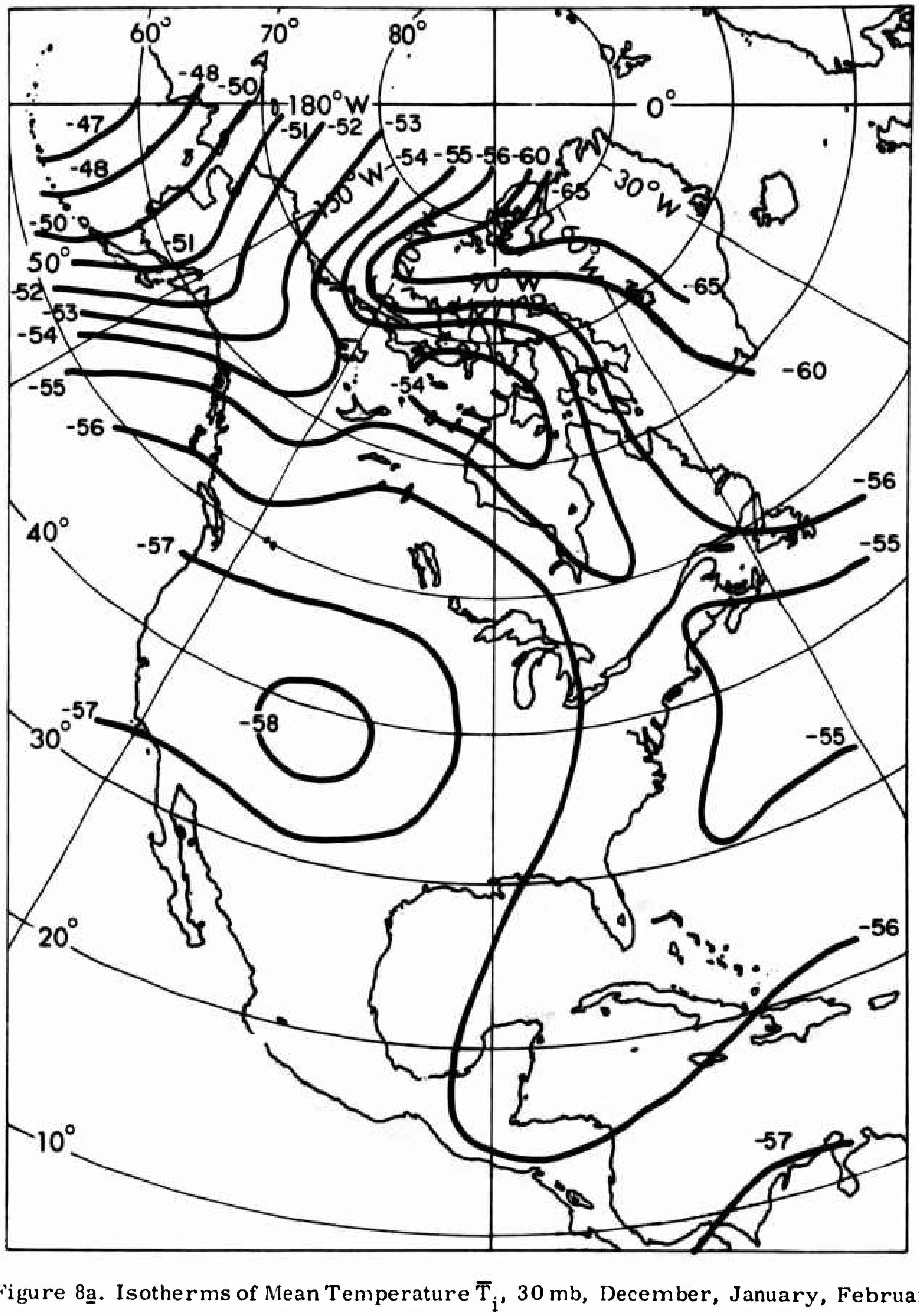

Figure 8a. Isotherms of Mean Temperature $\overline{T}_i$, 30 mb, December, January, February

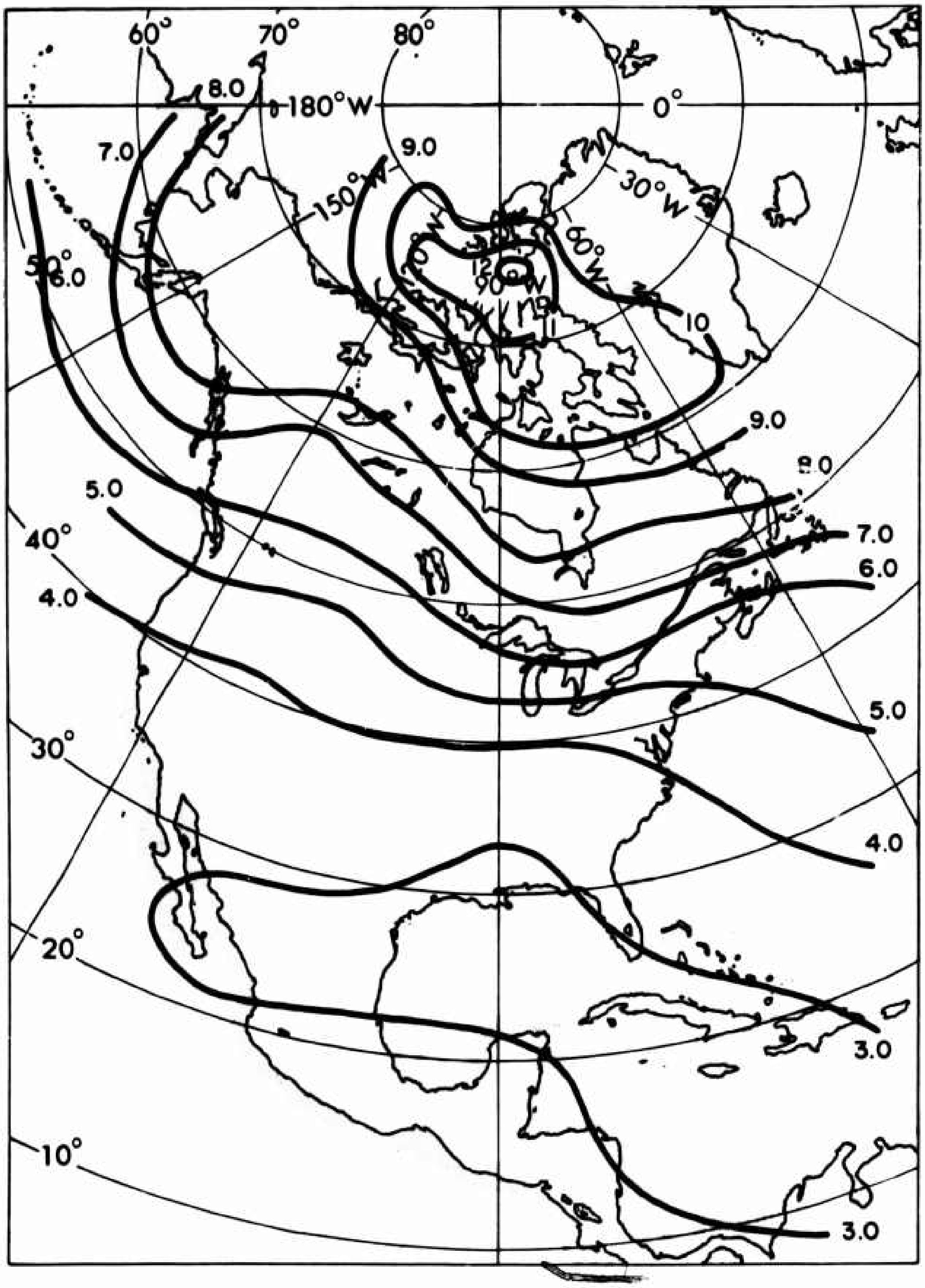

Figure 8b. Isopleths of Standard Deviation σ_i, 30 mb, December, January, February

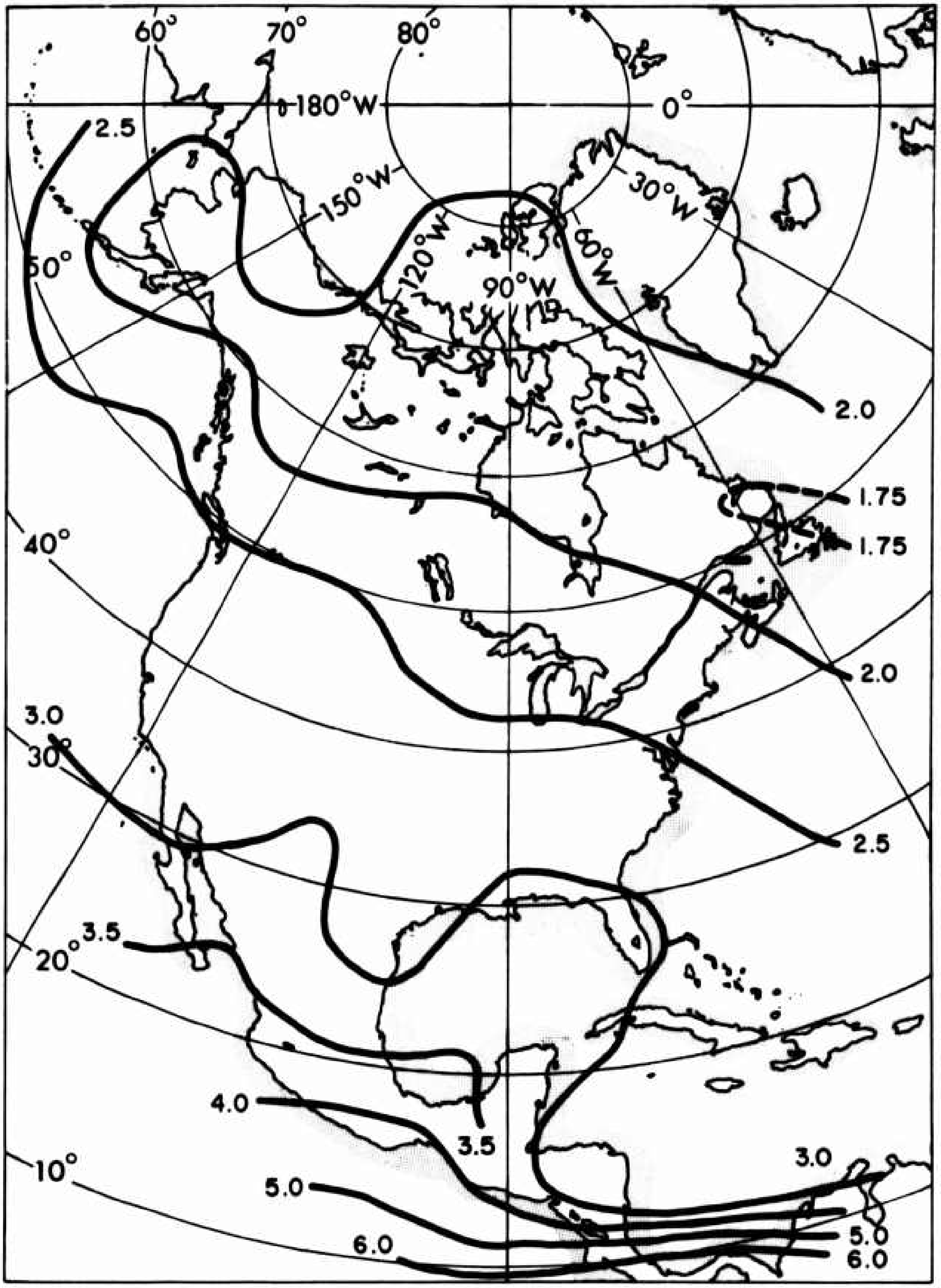

Figure 8c. Isopleths of Model Parameter α_1, 30 mb, December, January, February

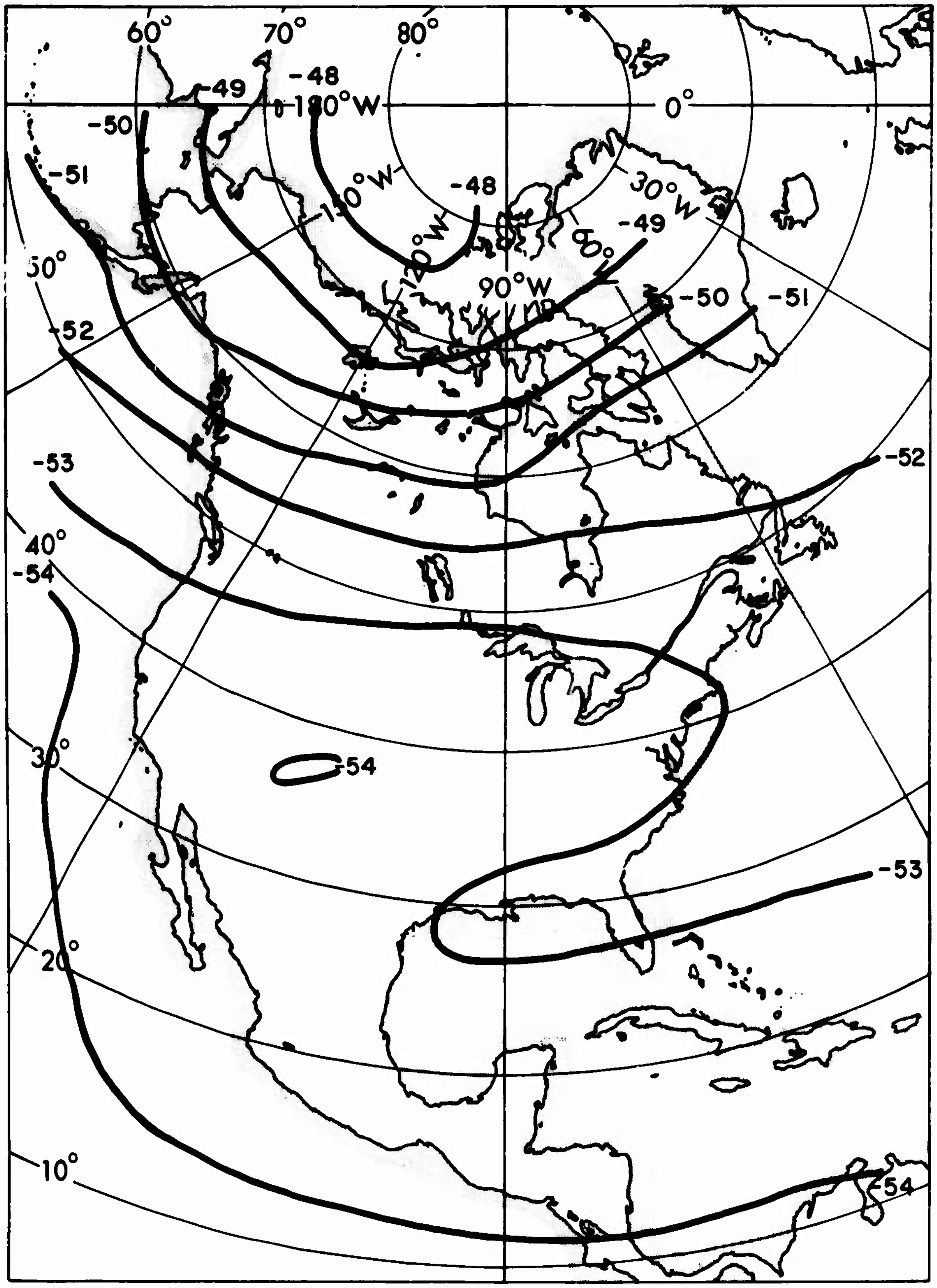

Figure 9a. Isotherms of Mean Temperature $\overline{T}_i$, 30 mb, March, April, May

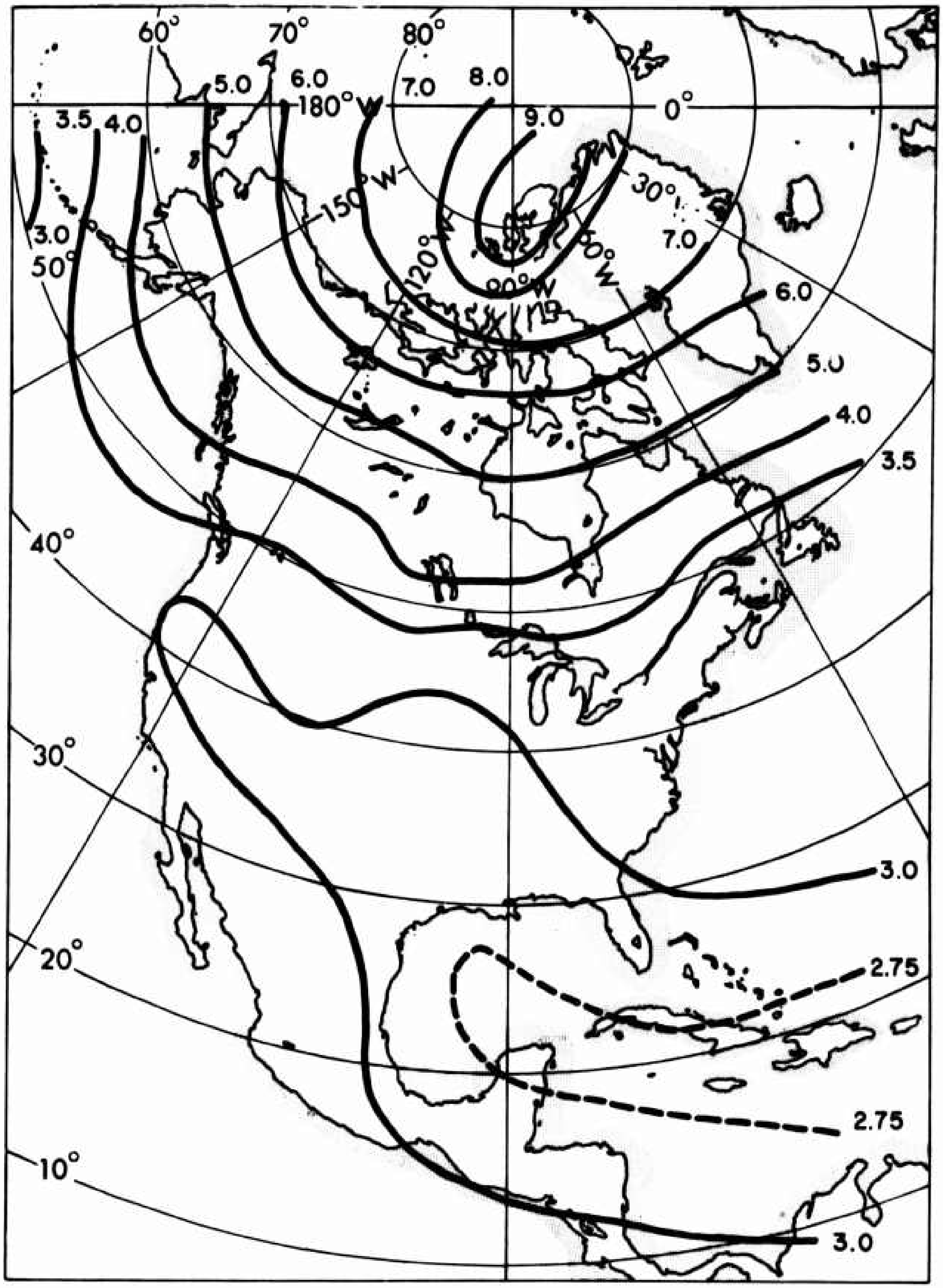

Figure 9b. Isopleths of Standard Deviation σ_i, 30 mb, March, April, May

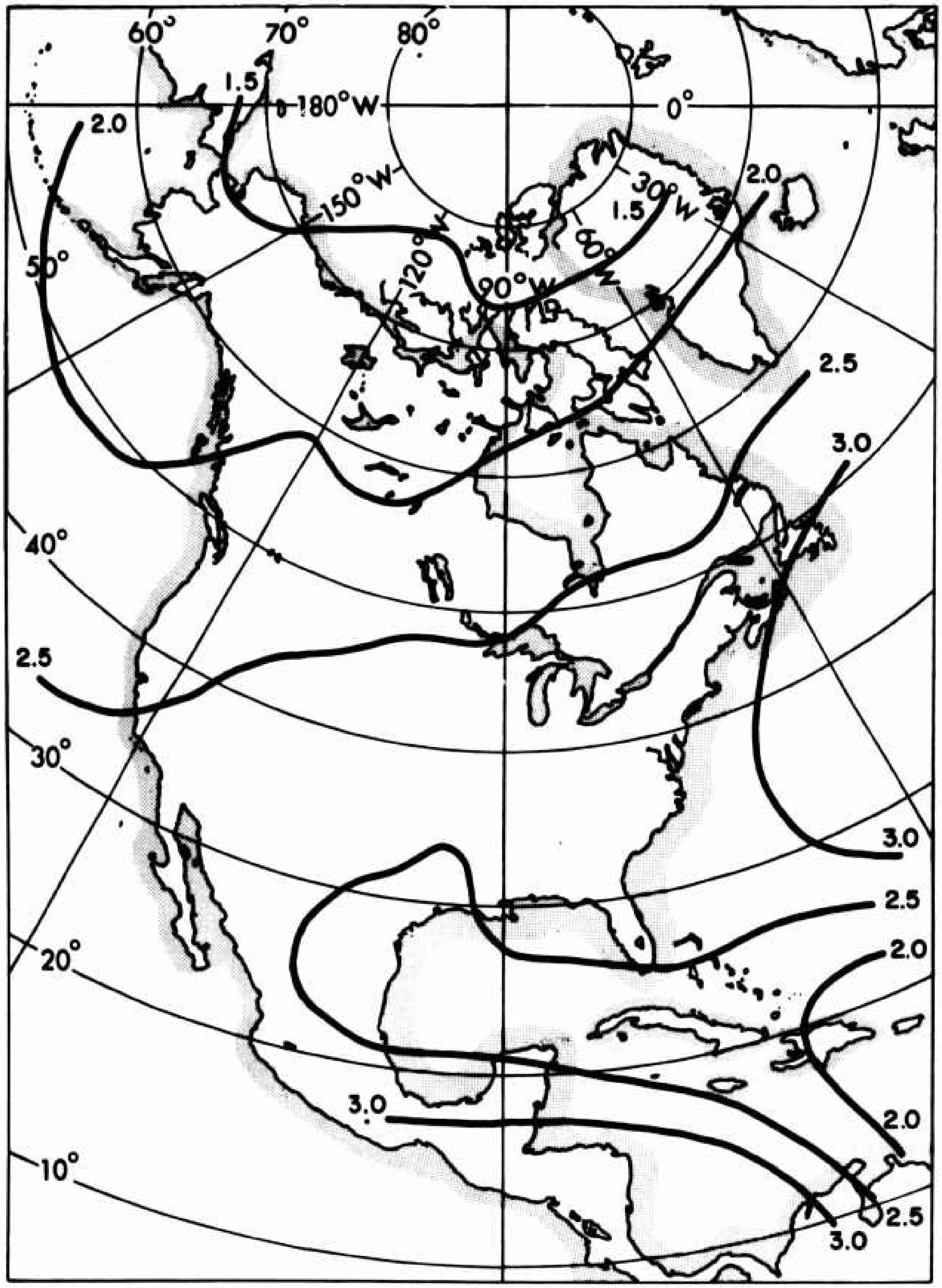

Figure 9c. Isopleths of Model Parameter α_i, 30 mb, March, April, May

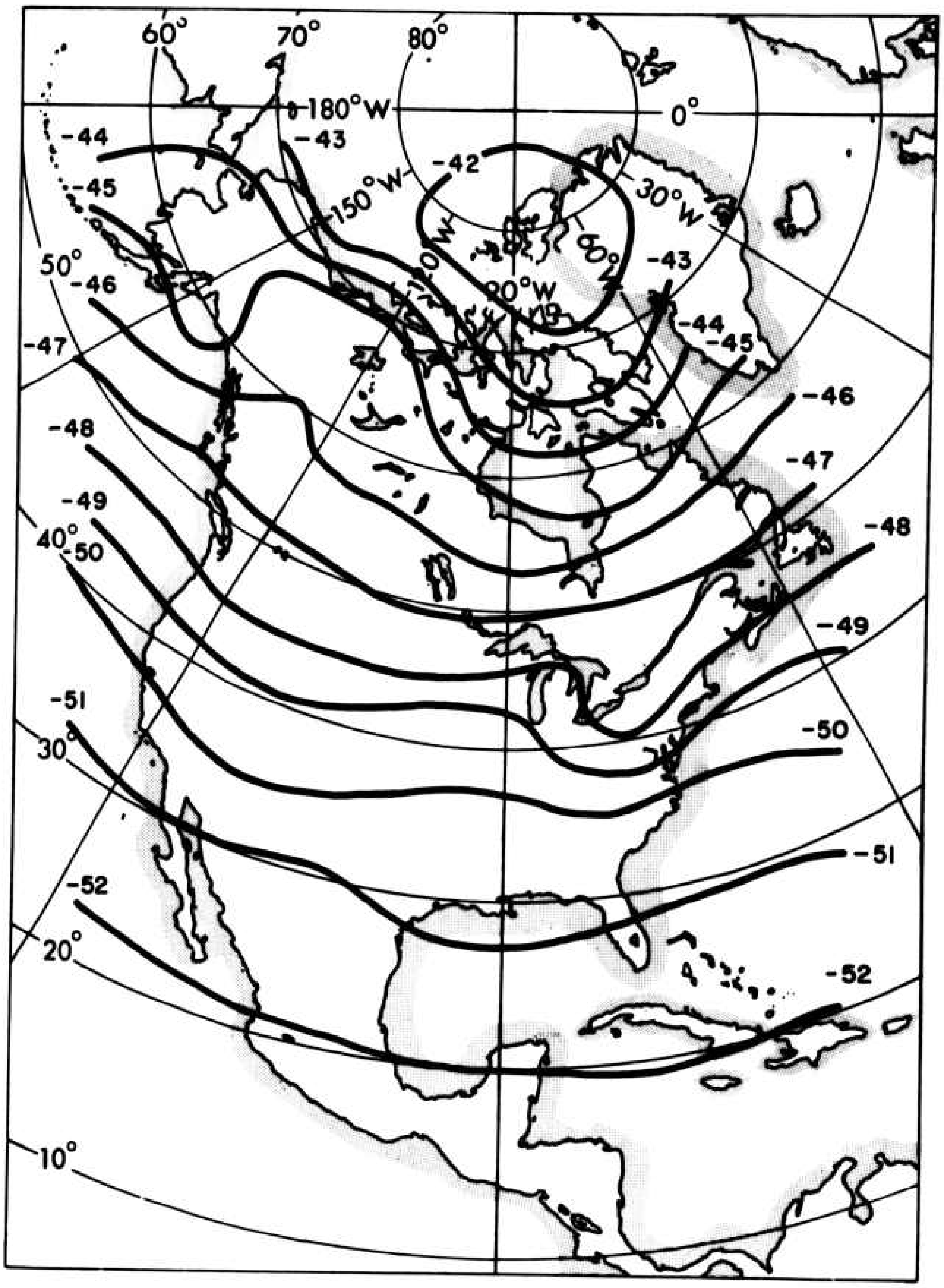

Figure 10a. Isotherms of Mean Temperature $\overline{T}_i$, 30 mb, June, July, August

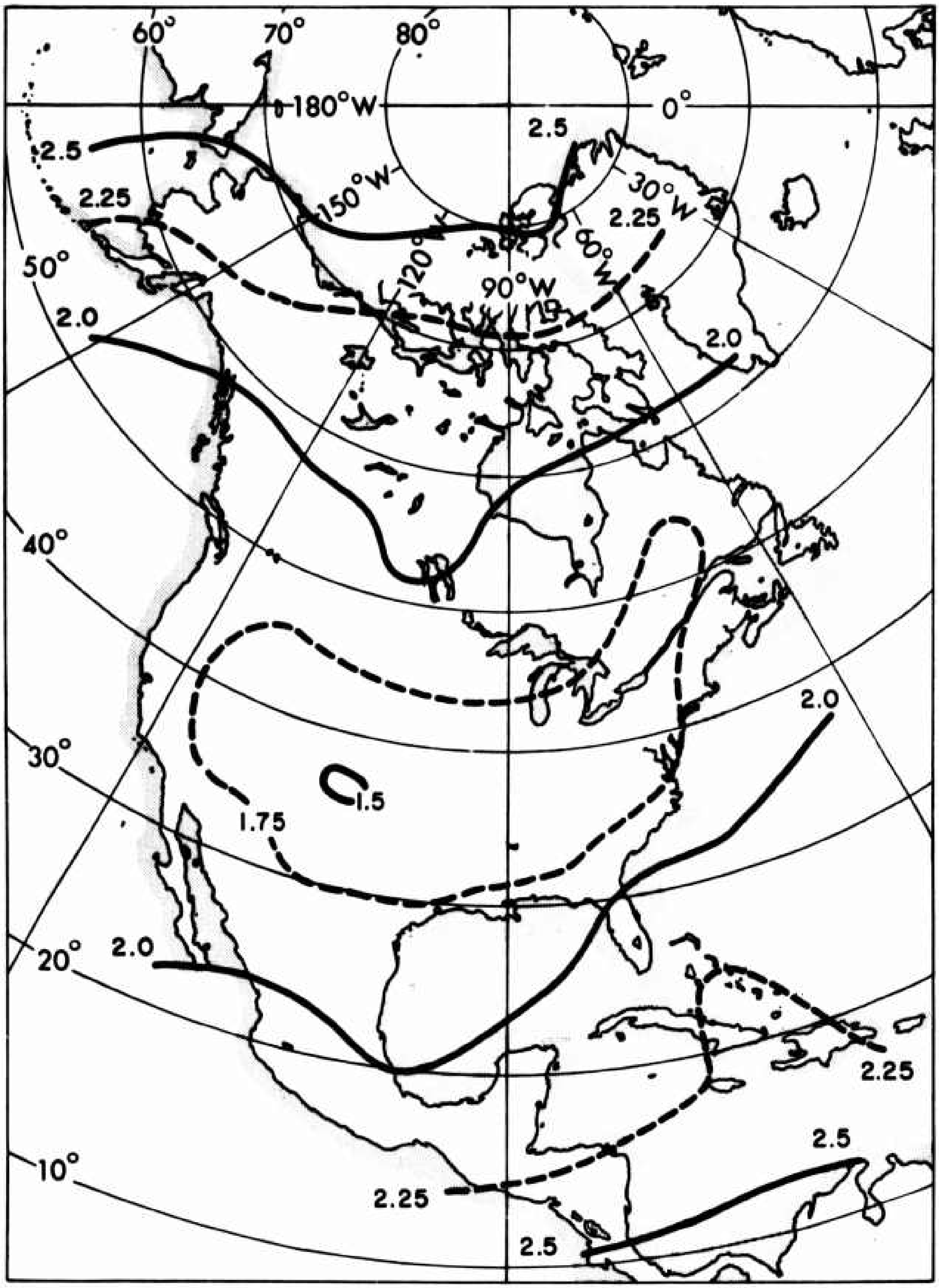

Figure 10b. Isopleths of Standard Deviation σ_i, 30 mb, June, July, August

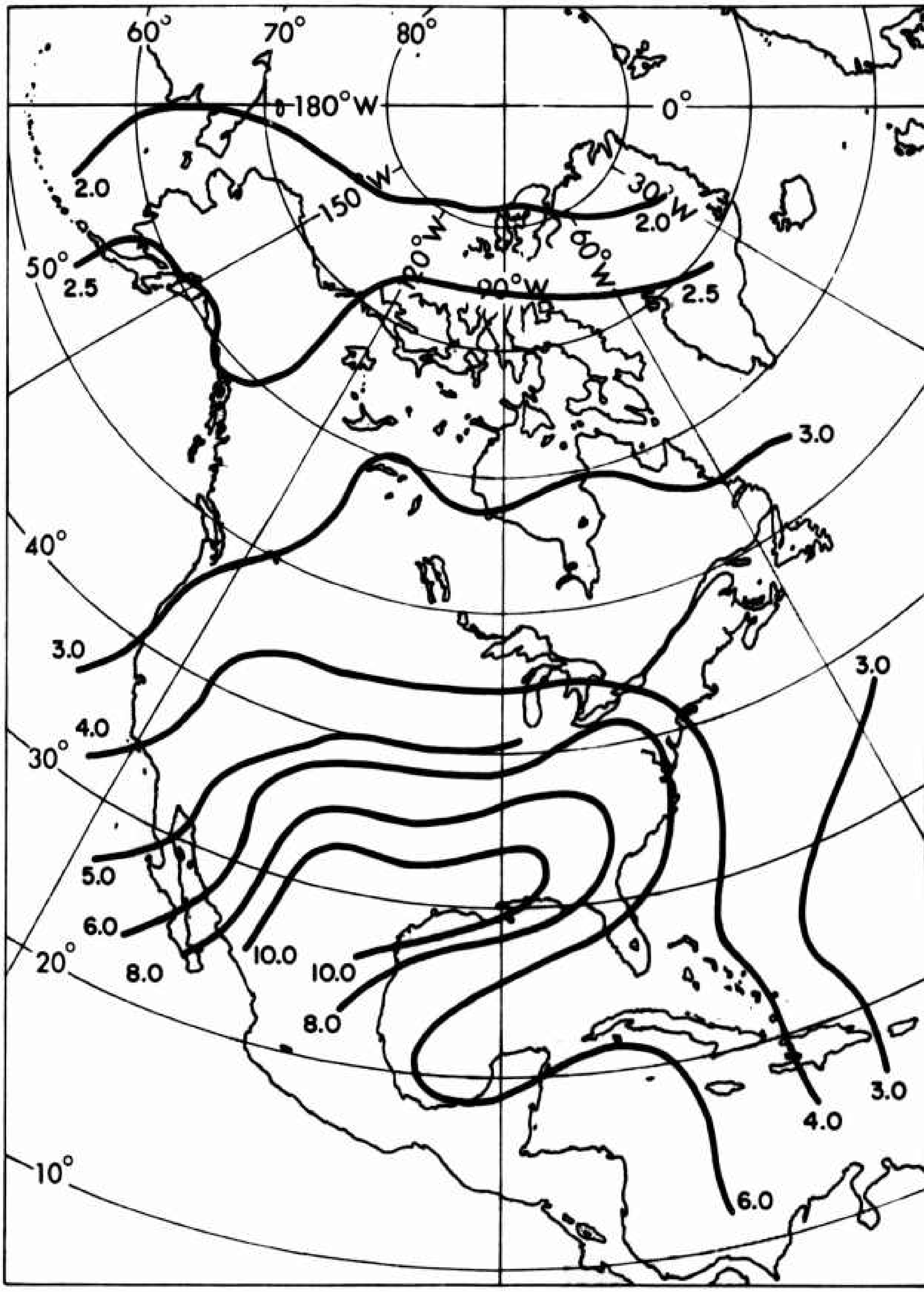

Figure 10c. Isopleths of Model Parameter α_i, 30 mb, June, July, August

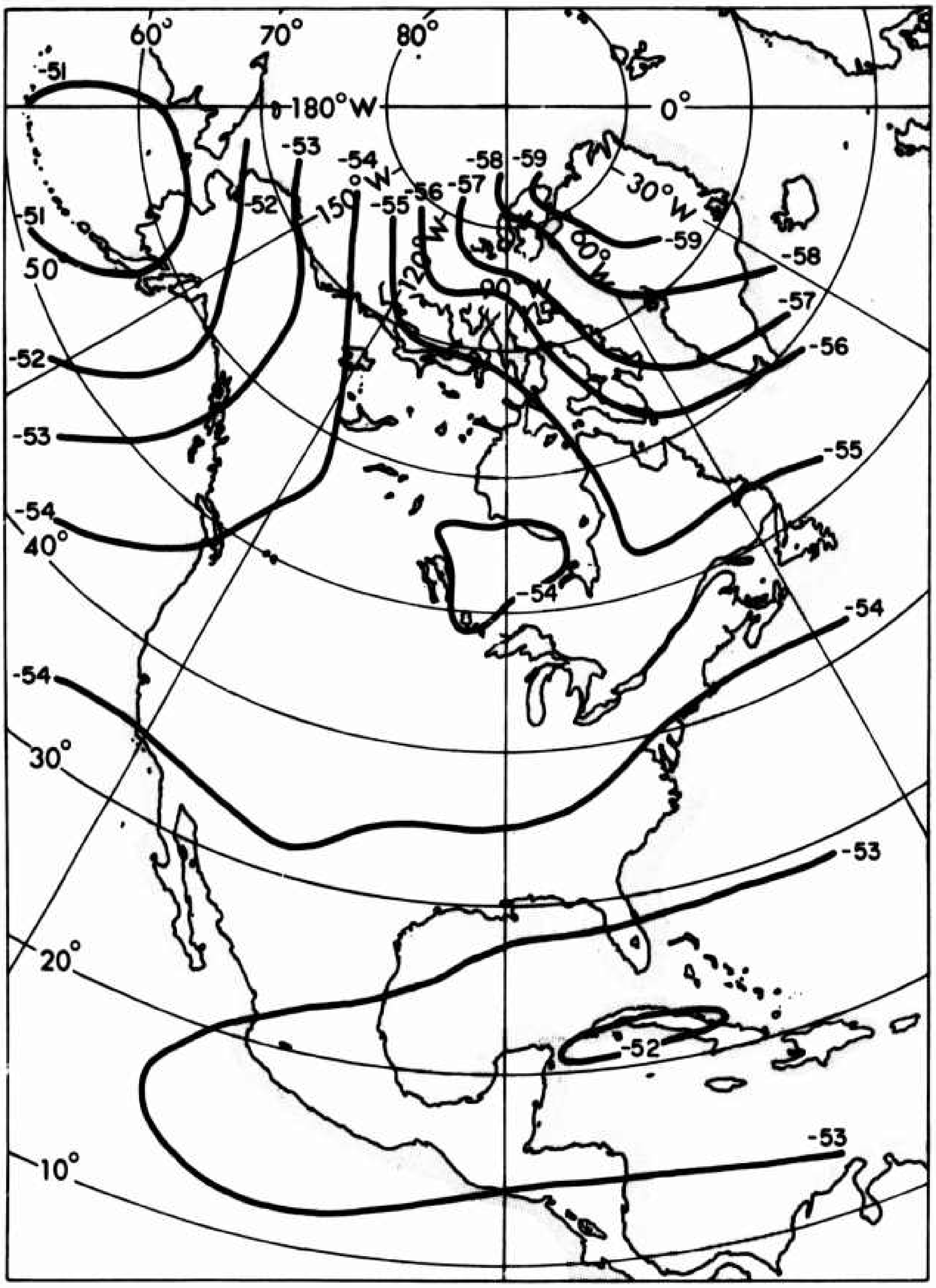

Figure 11a. Isotherms of Mean Temperature $\overline{T}_i$, 30 mb, September, October, November

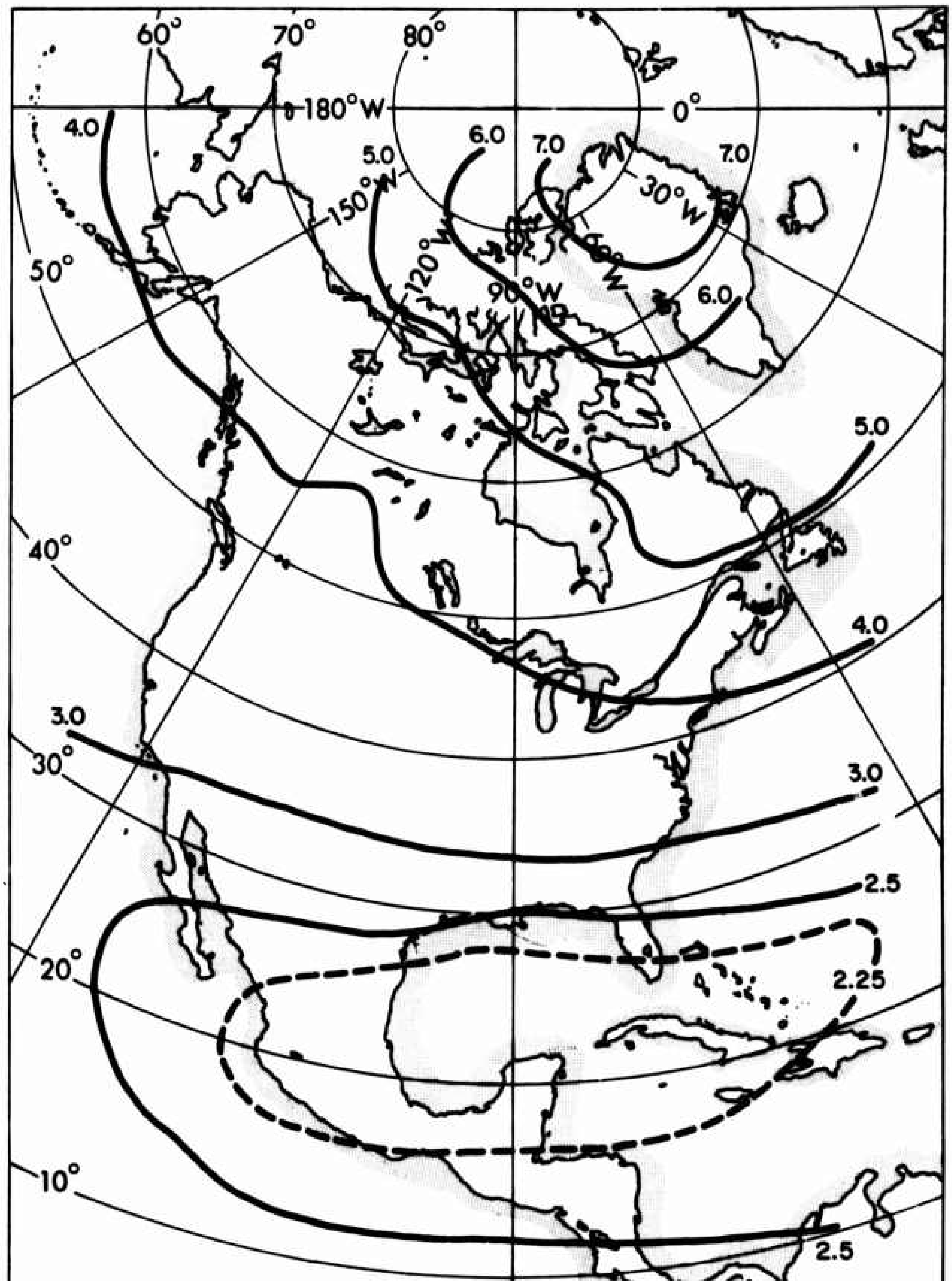

Figure 11b. Isopleths of Standard Deviation σ_i, 30 mb, September, October, November

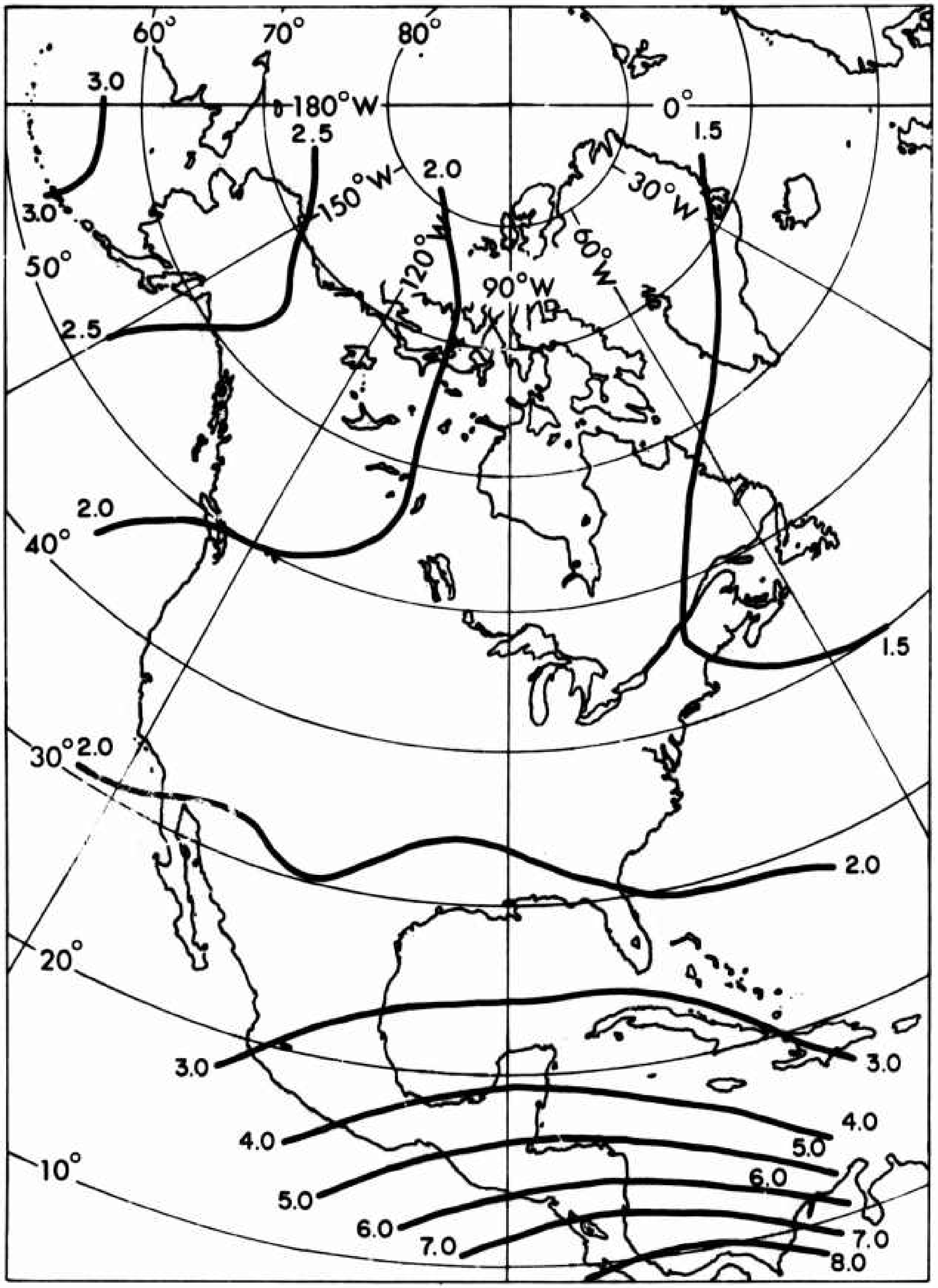

Figure 11c. Isopleths of Model Parameter α_i, 30 mb, September, October, November

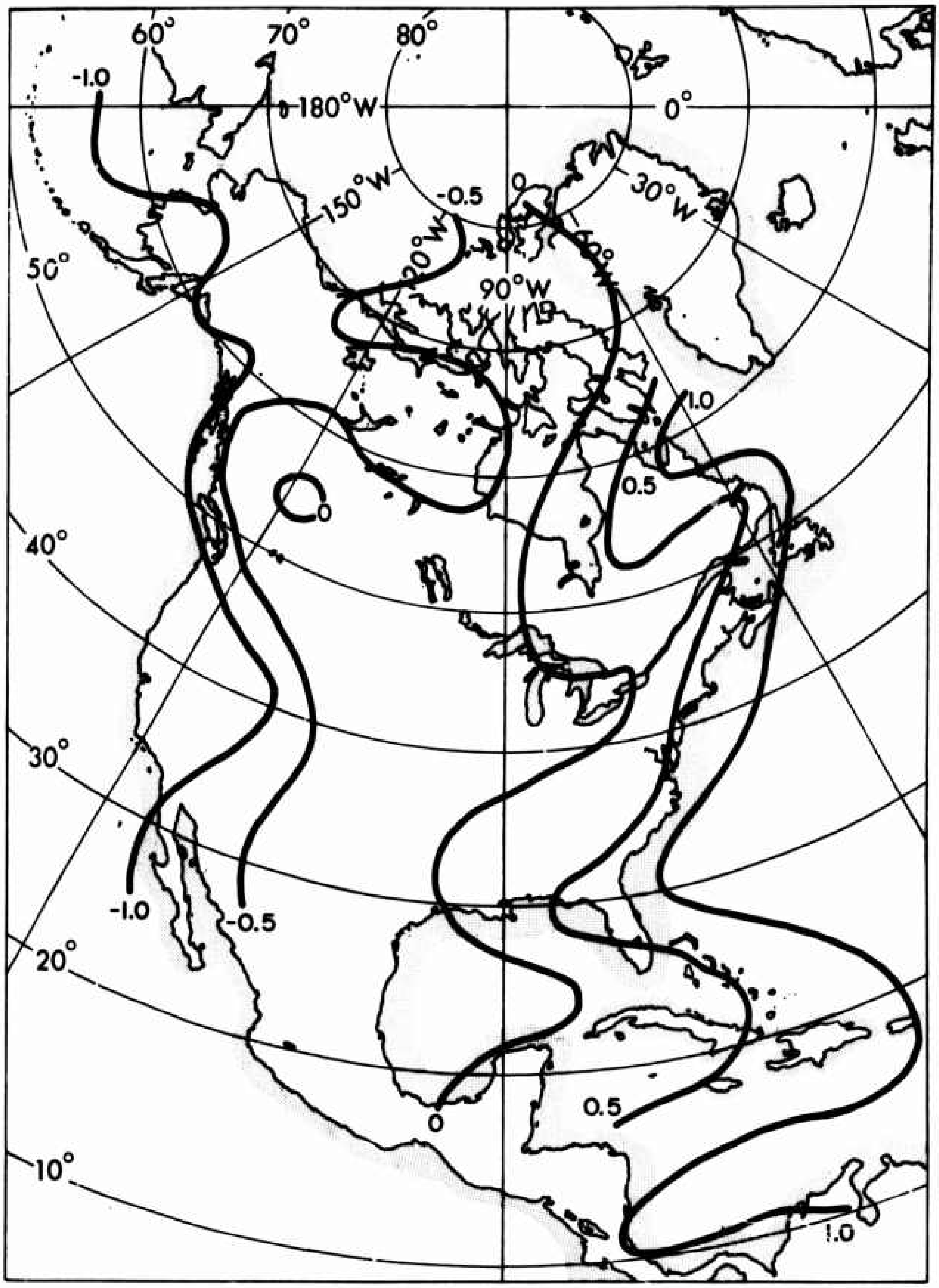

Figure 12. Isopleths of the Difference D Between Mean Temperatures, 30 mb, Winter, at Times 0000Z and 1200Z ($D = \overline{T}_{1200} - \overline{T}_{0000}$)

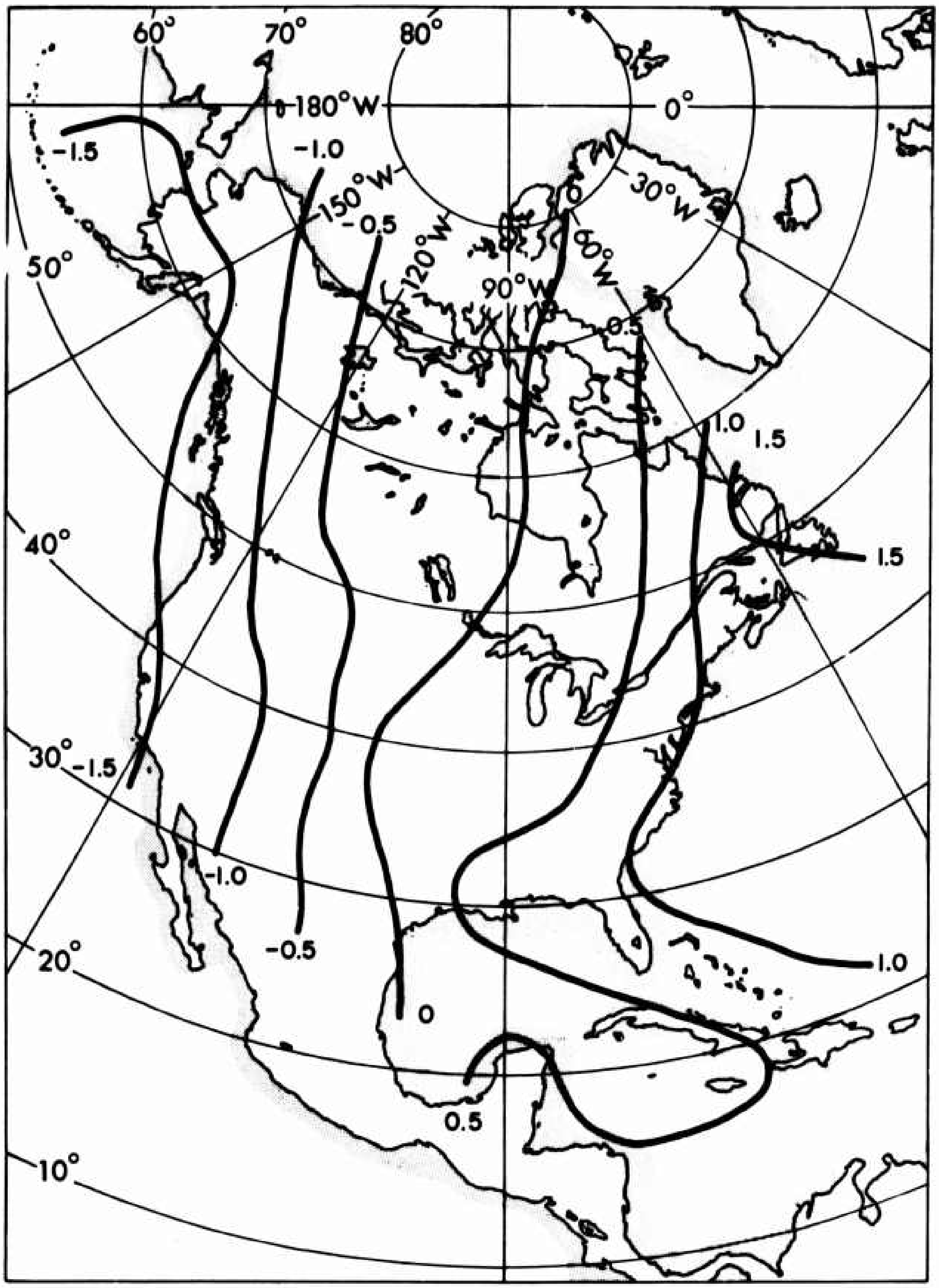

Figure 13. Isopleths of the Difference D Between Mean Temperatures, 30 mb, Summer, at Times 0000Z and 1200Z ($D = \overline{T}_{1200} - \overline{T}_{0000}$)

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Data archives are such that, for a selected cruising level, a reasonably large sample of daily average temperatures along a specific route can be obtained and summarized into a frequency distribution. But if, as in Air Force operations, many different and constantly changing routes are involved, then the climatologic tools and methods must be such as to provide estimates of the route average temperature without recourse to the archives and the processing of raw data. A method has been devised that utilizes a model of the horizontal spatial correlations of temperature. It has been demonstrated on a continent-wide area, and shows promise of hemisphere-wide application.

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